Section 31.9: Solutions of Nonhomogeneous Linear 2nd order DEs

Tuesday, February 28, 2017 2:30 PM

non-homogeneous:

$$ay'' + by' + cy = \underbrace{f(x)}_{\text{for homogeneous, this was zero}}$$

here's the plan:

1. solve the homogeneous case first

$$ay'' + by' + cy = 0$$

we call the solution y_c
(the complementary solution - it's the general solution to the associated homogeneous DE)

how? previous section

2. then find the particular solution necessary to get the right-hand-side $f(x)$ of our DE

how? this section

we call this solution y_p, the particular solution

③ $y = y_c + \underbrace{y_p}$

particular solution with <u>no</u> arbitrary constants c_1, c_2 etc

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so, how to do step 2? how to find y_p?

$$ay'' + by' + cy = f(x)$$

y_p is an expression that contains all possible forms of $f(x)$ and their derivatives

if $f(x) = x^2$, then $y_p = Ax^2 + Bx + C$

$f(x) = x^3$, then $y_p = Ax^3 + Bx^2 + Cx + D$

$f(x) = e^x$, then $y_p = Ae^x$

$f(x) = xe^x$, then $y_p = Axe^x + Be^x$

$f(x) = \sin x$, then $y_p = A\sin x + B\cos x$

(this is known as the method of undetermined coefficients) so, what do you do with this y_p once you have it?

plug it back into the DE to find the constents