Section 31.9: cont'd

Wednesday, March 1, 2017 1:33 PM

solve: $y'' + 9y = 8e^x$

step 1: solve $y'' + 9y = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$m = \pm 3i$

$m = a \pm bi$ where $a=0$

$y_c = e^{ax}(C_1 \cos bx + C_2 \sin bx)$

$y_c = \cancel{e^{0}}^{1} (C_1 \cos 3x + C_2 \sin 3x)$

$\boxed{y_c = C_1 \cos 3x + C_2 \sin 3x}$

step 2: $y'' + 9y = 8e^x$

$y_p = Ae^x$

now substitute into OE:

$y_p = Ae^x$

$y_p' = Ae^x$

$y_p'' = Ae^x$

$y'' + 9y = 8e^x$

$Ae^x + 9Ae^x = 8e^x$

$10Ae^x = 8e^x$

$$10A = 8$$

$$A = 4/5$$

so $\boxed{y_p = 4/5\, e^x}$

step 3: $y = y_c + y_p$

$$= C_1 \cos 3x + C_2 \sin 3x + 4/5\, e^x$$

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example: solve $y'' + 4y = 2\sin 3x$

complementary: $y'' + 4y = 0$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$

particular: $y'' + 4y = 2\sin 3x$

$$y_p = A \sin 3x + B\cos 3x$$

$$y_p' = 3A\cos 3x - 3B\sin 3x$$

$$y_p'' = -9A\sin 3x - 9B\cos 3x$$

now plug into DE:

$$y'' + 4y = 2\sin 3x$$

$$-9A\sin 3x - 9B\cos 3x + 4A\sin 3x + 4B\cos 3x = 2\sin 3x$$

$$-5A\sin 3x - 5B\cos 3x = 2\sin 3x + 0\cos 3x$$

then $-5A = 2$ and $-5B = 0$

$A = -2/5$ $B = 0$

$$y_p = -\frac{2}{5}\sin 3x$$

full solution:

$$y = y_c + y_p$$

$$= C_1\cos 2x + C_2\sin 2x - \frac{2}{5}\sin 3x$$

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solve: $2y'' + 5y' - 3y = e^x + 4e^{2x}$

complementary sol'n:

$$2m^2 + 5m - 3 = 0$$
$$2m^2 + 6m - m - 3 = 0$$
$$2m(m+3) - 1(m+3) = 0$$
$$(2m-1)(m+3) = 0$$
$$m = 1/2, -3$$

$ac = -6$

$-1 + 6$

$2 \; 3$

$$y_c = C_1 e^{\frac{1}{2}x} + C_2 e^{-3x}$$

particular solution

$RHS = e^x + 4e^{2x}$

$$y_p = Ae^x + Be^{2x}$$

$$y_p = Ae^{x} + Be^{2x}$$

$$y_p' = Ae^{x} + 2Be^{2x}$$

$$y_p'' = Ae^{x} + 4Be^{2x}$$

substitute in:

$$2y'' + 5y' - 3y = e^{x} + 4e^{2x}$$

$$2(Ae^{x} + 4Be^{2x}) + 5(Ae^{x} + 2Be^{2x}) - 3(Ae^{x} + Be^{2x}) = e^{x} + 4e^{2x}$$

$$2Ae^{x} + 8Be^{2x} + 5Ae^{x} + 10Be^{2x} - 3Ae^{x} - 3Be^{2x} = e^{x} + 4e^{2x}$$

$$4Ae^{x} + 15Be^{2x} = e^{x} + 4e^{2x}$$

so $4A = 1$ and $15B = 4$

$A = 1/4$ $\quad B = 4/15$

$$y_p = \tfrac{1}{4}e^{x} + \frac{4}{15}e^{2x}$$

full solution:

$$y = y_c + y_p$$

$$= c_1 e^{\frac{1}{2}x} + c_2 e^{-3x} + \frac{1}{4}e^{x} + \frac{4}{15}e^{2x}$$