Section 31.9: cont'd

Thursday, March 2, 2017 1:34 PM

solve: $y'' + 4y = x - 4e^{-x}$

Complementary solution: $y'' + 4y = 0$

$m^2 + 4 = 0$

$m = \pm 2i$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

particular solution: RHS = $x - 4e^{-x}$

$y_p = Ax + B + Ce^{-x}$

$y_p' = A - Ce^{-x}$

$y_p'' = + Ce^{-x}$

$y'' + 4y = x - 4e^{-x}$

$Ce^{-x} + 4(Ax + B + Ce^{-x}) = x - 4e^{-x}$

$4Ax + 4B + Ce^{-x} + 4Ce^{-x} = x - 4e^{-x}$

? zero

$4Ax + 4B + 5Ce^{-x} = x - 4e^{-x} + \bigcirc$

so $4A = 1$ and $4B = 0$ and $5C = -4$

$A = 1/4$ $\quad B = 0$ $\quad C = -4/5$

$y_p = \frac{1}{4}x - \frac{4}{5}e^{-x}$

full solution:

$$y = y_c + y_p$$

$$= C_1 \cos 2x + C_2 \sin 2x + \tfrac{1}{4}x - \tfrac{4}{5}e^{-x}$$

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the "bad case":

example: solve

$$y'' - 5y' + 4y = 12e^x$$

complementary solution: $y'' - 5y' + 4y = 0$

$$m^2 - 5m + 4 = 0$$

$$(m-1)(m-4) = 0$$

$$m = 1, 4$$

$$y_c = C_1 e^x + C_2 e^{4x}$$

particular solution: $\text{RHS} = 12e^x$

$$y_p = \cancel{Ae^x}$$

like terms!

bad case!

~~why won't this work?~~

$$y_p = Ae^x$$

$$y_p' = Ae^x$$

$$y_p'' = Ae^x$$

$$\cancel{y'' - 5y' + 4y = 12e^x}$$

$$\cancel{Ae^x - 5Ae^x + 4Ae^x = 12e^x}$$

$$\cancel{0 = 12e^x}$$

"bad case": when your initial guess for y_p has "like terms" to y_c

$y_p = Axe^x$

take the original y_p and multiply by x

$$y_p = Axe^x$$

$$y_p' = Axe^x + Ae^x$$

$$y_p'' = Axe^x + Ae^x + Ae^x$$

$$= Axe^x + 2Ae^x$$

now substitute:

$$y'' - 5y' + 4y = 12e^x$$

$$(Axe^x + 2Ae^x) - 5(Axe^x + Ae^x) + 4Axe^x = 12e^x$$

$$Axe^x + 2Ae^x - 5Axe^x - 5Ae^x + 4Axe^x = 12e^x$$

$$\underbrace{(Axe^x - 5Axe^x + 4Axe^x)}_{0} + \underbrace{(2Ae^x - 5Ae^x)} = 12e^x$$

$$-3Ae^x = 12e^x$$

so $-3A = 12$

$A = -4$

$$y_p = -4xe^x$$

full solution: $y = y_c + y_p$

$$= c_1 e^{x} + c_2 e^{4x} - 4xe^{x}$$

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example: solve: $y'' + 9y = 4 \sin 3x$

complementary solution: $y'' + 9y = 0$

$$m^2 + 9 = 0$$
$$m = \pm 3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x$$

particular solution: RHS $= 4 \sin 3x$

BAD CASE!

$$y_p = x(A \sin 3x + B \cos 3x)$$