Section 3.9: cont'd

Friday, March 3, 2017 2:45 PM

recall from last time,

solve $y'' + 9y = 4\sin 3x$

we found $y_c = C_1 \cos 3x + C_2 \sin 3x$

particular solution: $RHS = 4\sin 3x$

$$y_p = \cancel{A\sin 3x} + \cancel{B\cos 3x}$$

$$= x(A\sin 3x + B\cos 3x)$$

$$= Ax\sin 3x + Bx\cos 3x$$

$$y_p = x(A\sin 3x + B\cos 3x)$$

$$y_p' = (A\sin 3x + B\cos 3x) + x(3A\cos 3x - 3B\sin 3x)$$

$$y_p'' = 3A\cos 3x - 3B\sin 3x + (3A\cos 3x - 3B\sin 3x)$$

$$+ x(-9A\sin 3x - 9B\cos 3x)$$

$$= 6A\cos 3x - 6B\sin 3x - 9x(A\sin 3x + B\cos 3x)$$

$$y'' + 9y = 4\sin 3x$$

$$6A\cos 3x - 6B\sin 3x - \cancel{9x(A\sin 3x + B\cos 3x)}$$

$$+ \cancel{9x(A\sin 3x + B\cos 3x)} = 4\sin 3x$$

$$6A\cos 3x - 6B\sin 3x = 4\sin 3x$$

so $-6B = 4$, $B = -2/3$ and $6A = 0$, $A = 0$

$$y_p = -\frac{2}{3}x\cos 3x$$

full solution:

$$y = y_c + y_p$$

$$= C_1\cos 3x + C_2\sin 3x - \tfrac{2}{3}x\cos 3x$$

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example: solve:

$$y'' - 2y' + y = xe^{2x} - e^{2x}$$

given that $y' = 4$ and $y = -2$ when $x = 0$

complementary solution: $m^2 - 2m + 1 = 0$

$$m = 1$$

$$y_c = (C_1 + C_2 x)e^{x}$$

particular solution: RHS: $xe^{2x} - e^{2x}$

$$y_p = Axe^{2x} + Be^{2x}$$

$$y_p' = 2Axe^{2x} + Ae^{2x} + 2Be^{2x}$$

$$y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} + 4Be^{2x}$$

$$y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} + 4Be^{2x}$$

$$= 4Axe^{2x} + 4Ae^{2x} + 4Be^{2x}$$

plug back in:

$$y'' - 2y' + y = xe^{2x} - e^{2x}$$

$$(4Axe^{2x} + 4Ae^{2x} + 4Be^{2x}) - 2(2Axe^{2x} + Ae^{2x} + 2Be^{2x})$$

$$+ Axe^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$\cancel{4Axe^{2x}} + 4Ae^{2x} + \cancel{4Be^{2x}} - \cancel{4Axe^{2x}} - 2Ae^{2x} - \cancel{4Be^{2x}}$$

$$+ Axe^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$Axe^{2x} + 2Ae^{2x} + Be^{2x} = xe^{2x} - e^{2x}$$

$$Axe^{2x} + (2A+B)e^{2x} = xe^{2x} - e^{2x}$$

so $A = 1$ and $2A + B = -1$

$$2 \cdot 1 + B = -1$$

$$B = -3$$

$$y_p = xe^{2x} - 3e^{2x}$$

full solution: $y = y_c + y_p$

$$= (c_1 + c_2 x)e^{x} + xe^{2x} - 3e^{2x}$$

so, initial conditions: when $x = 0$, $y = -2$

$$-2 = (c_1 + 0) + 0 - 3$$

$c_1 = 1$

when $x = 0$, $y' = 4$

$$y = (1 + c_2 x)e^x + xe^{2x} - 3e^{2x}$$

$$y' = c_2 e^x + (1 + c_2 x)e^x + 2xe^{2x} + e^{2x} - 6e^{2x}$$

$$4 = c_2 + 1 + 0 + 1 - 6$$

$$c_2 = 8$$

$$y = (1 + 8x)e^x + (x - 3)e^{2x}$$