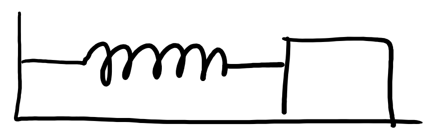
Section 31.10: cont'd

Tuesday, March 7, 2017 2:01 PM

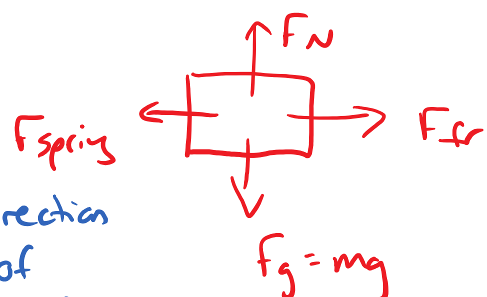
back to Hooke's Law:

what if there is air resistance?

$$\vec{F}_{fr} = -b\vec{v}$$



free-body diagram:



direction of force depends on x

direction of air resistance depends of velocity

$$\Sigma F = m\vec{a}$$

$$F_{spring} - F_{fr} = ma$$

$$-kx - bv = ma$$

$$0 = ma + bv + kx$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$(m\ddot{x} + b\dot{x} + kx = 0)$$

auxiliary equation:

$$mn^2 + bn + k = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

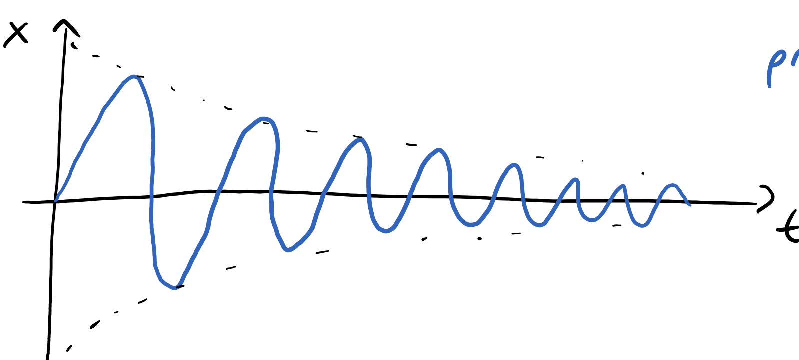
solutions will be

① 2 distinct real if $b^2 - 4km > 0$
② 1 repeated real " $= 0$
③ 2 complex " < 0

① $x_1 = c_1 e^{n_1 t} + c_2 e^{n_2 t}$
② $x_2 = (c_1 + c_2 t) e^{nt}$
③ $x_3 = e^{at} (c_1 \cos bt + c_2 \sin bt)$

what do these solutions look like?

③



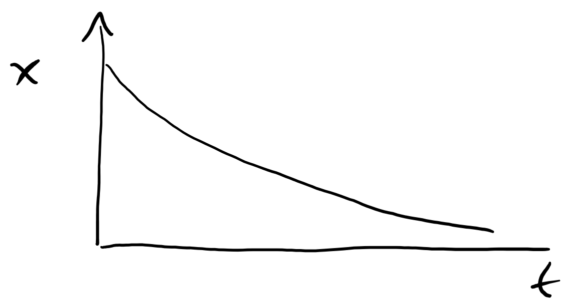
provided a is negative, so e^{at} is exponential decay

sinusoidal with

sinusoidal with
decaying amplitude

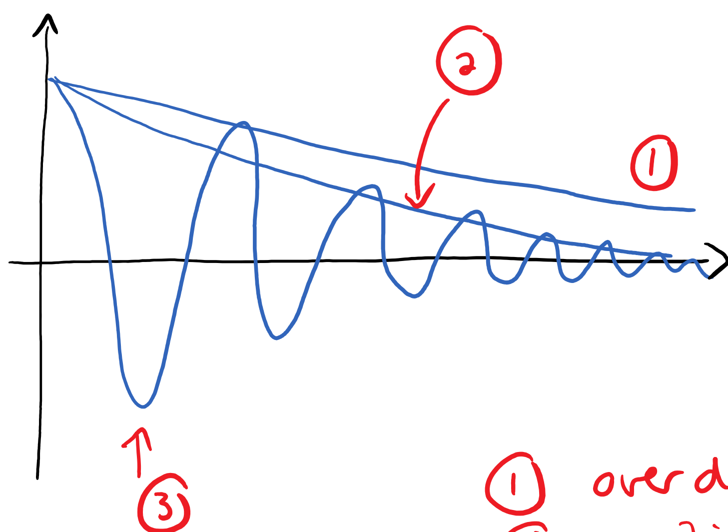
but what do ① and ② look like?

if the r's (solns to the aux eqn) are negative



exponential decay

so putting it all together:



critically damped - just enough friction to prevent oscillation

→ object "returns to equilibrium" in minimum time

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so, what about an external force?

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = \underbrace{F_{ext}(t)}_{}$$

$F_{external}$ is a function of time

note: resonance happens when your external force is pumping with a frequency that approaches the natural frequency of the oscillator

→ amplitude of oscillation increases exponentially

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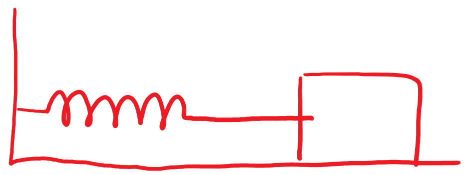
example: consider a mass on a spring on a frictionless surface, but do not neglect air resistance

a) write the differential equation if there are no external forces acting on the mass

b) if $m = 5\,kg$ and $k = 30\ N/m$, what value of b leads to critical damping (don't worry about units)

a)



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

b)

$$mn^2 + bn + k = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

critically damped is the case with 1 real solution

so $b^2 - 4km = 0$

$$b^2 = 4km$$

$$b = \pm\sqrt{4km}$$

$$= \sqrt{4(30)(5)}$$

$$= \sqrt{600}$$
$$= 10\sqrt{6}$$
$$\approx 24.5$$

nitpicker note: what <u>are</u> the units?

$$\underbrace{m\frac{d^2x}{dt^2}}_{kg\,\frac{m}{s^2}} + \underbrace{b\frac{dx}{dt}}_{\boxed{}\,m/s} + \underbrace{kx}_{\frac{N}{m}\cdot m} = 0$$

↑
must be
kg/s

$$b \sim \sqrt{km}$$

$$\sim \sqrt{\frac{kg\,m}{m\,s^2} \cdot kg}$$