Tutorial

Thursday, March 9, 2017 12:38 PM

solve: $y'' - y' - 6y = 4x$

complementary:

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$m = 3, -2$

$y_c = C_1 e^{3x} + C_2 e^{-2x}$

particular: RHS = $4x$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$y'' - y' - 6y = 4x$

$0 - A - 6(Ax + B) = 4x$

$-A - 6Ax - 6B = 4x$

$(-6A)x + (-A - 6B) = 4x + 0$

so $-6A = 4$ and $-A - 6B = 0$

$A = -2/3$ $\quad$ $+2/3 - 6B = 0$

$6B = 2/3$

$B = 1/9$

$y_p = -\frac{2}{3}x + \frac{1}{9}$

full solution: $y = y_c + y_p$

$$= C_1 e^{3x} + C_2 e^{-2x} - \frac{2}{3}x + 1/9$$

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solve: $y'' + 4y' + 3y = 2 + e^x$

complementary solution:

$$m^2 + 4m + 3 = 0$$
$$(m+1)(m+3) = 0$$
$$m = -1, -3$$

$$y_c = C_1 e^{-x} + C_2 e^{-3x}$$

particular solution: RHS = $2 + e^x$

$$y_p = A + Be^x$$
$$y_p' = Be^x$$
$$y_p'' = Be^x$$

$$y'' + 4y' + 3y = 2 + e^x$$

$$Be^x + 4Be^x + 3(A + Be^x) = 2 + e^x$$
$$3A + 8Be^x = 2 + e^x$$

so $3A = 2$ and $8B = 1$

$A = 2/3$ $\quad B = 1/8$

$$y_p = \frac{2}{3} + \frac{1}{8}e^x$$

full: $y = y_c + y_p = C_1 e^{-x} + C_2 e^{-3x} + \frac{2}{3} + \frac{1}{8}e^x$

$$y'' - y = e^{-x}$$

complementary solution: $m^2 - 1 = 0$

$m = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

particular solution: RHS: e^{-x} BAD CASE!

$y_p = Axe^{-x}$

so $y_p = Axe^{-x}$

$y_p' = Ae^{-x} + -Axe^{-x}$

$y_p'' = -Ae^{-x} - Ae^{-x} + Axe^{-x}$

$= -2Ae^{-x} + Axe^{-x}$

sub in:

$$y'' - y = e^{-x}$$

$$-2Ae^{-x} + \cancel{Axe^{-x}} - \cancel{Axe^{-x}} = e^{-x}$$

$$-2Ae^{-x} = e^{-x}$$

so $-2A = 1$

$A = -\frac{1}{2}$

$y_p = -\frac{1}{2}xe^{-x}$

full: $y = y_c + y_p$

$= c_1 e^{x} + c_2 e^{-x} - \frac{1}{2}xe^{-x}$

Solve $y'' - 2y' + y = 3\sin x$

Complementary:

$$m^2 - 2m + 1 = 0$$
$$(m-1)^2 = 0$$
$$m = 1$$
(repeated root)

$$y_c = (c_1 + c_2 x)e^x$$

particular: RHS: $3\sin x$

$$y_p = A\sin x + B\cos x$$
$$y_p' = A\cos x - B\sin x$$
$$y_p'' = -A\sin x - B\cos x$$

$$y'' - 2y' + y = 3\sin x$$

$$(-A\sin x - B\cos x) - 2(A\cos x - B\sin x) + (A\sin x + B\cos x) = 3\sin x$$

$$-2A\cos x + 2B\sin x = 3\sin x$$

so $-2A = 0$ and $2B = 3$
$A = 0$ $\quad B = 3/2$

$$y_p = \tfrac{3}{2}\cos x$$

full: $y = y_c + y_p = (c_1 + c_2 x)e^x + \tfrac{3}{2}\cos x$