

Section 3: Discrete Random Variables

Monday, March 20, 2017 1:17 PM

definition: a discrete random variable is a variable that can only take on certain values

example: consider the number of heads observed in 3 fair coin tosses

let $x =$ the number of heads observed

x	outcomes	# outcomes	$p(x)$
0	TTT	1	$\frac{1}{8}$
1	HTT THT TTH	3	$\frac{3}{8}$
2	HHT HTH THH	3	$\frac{3}{8}$
3	HHH	1	$\frac{1}{8}$
		8	

definition: the probability distribution of a discrete random variable is a table (or formula or graph) listing the probability for each possible value of the random variable x

so for our previous example, the probability distribution is

x	p(x)
0	1/8
1	3/8
2	3/8
3	1/8

so, how can we find the mean? the std dev?

the mean is

$$\mu = \sum x p(x)$$

this is also called the expected value $E(x)$

the variance

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

the std dev

$$\sigma = \sqrt{\sigma^2}$$

handout question 1 section 3

x	P(x)
-5	0.15
-2	0.2
1	0.4
6	0.25

$$\begin{aligned}
 \text{a) } P(-2.5 < x < 2.5) &= P(-2) + P(1) \\
 &= 0.2 + 0.4 \\
 &= 0.6
 \end{aligned}$$

b) the mean of x:

$$\begin{aligned}
 \mu &= \sum x p(x) \\
 &= (-5)(0.15) + (-2)(0.2) + 1(0.4) \\
 &\quad + 6(0.25) \\
 &= 0.75
 \end{aligned}$$

c) the variance of x:

$$\begin{aligned}
 \sigma^2 &= \sum x^2 p(x) - \mu^2 \\
 &= (-5)^2(0.15) + (-2)^2(0.2) + (1)^2(0.4) + 6^2(0.25) \\
 &\quad - (0.75)^2 \\
 &= 13.3875
 \end{aligned}$$

d) the std dev:

$$\sigma = \sqrt{\sigma^2} = \sqrt{13.3875} \approx 3.66$$

e) the probability that an x-value lies

within one standard deviation of the mean

$$P(\mu - \sigma \leq x \leq \mu + \sigma) = P(-2.91 \leq x \leq 4.41)$$

one std
dev below
mean

$$= P(-2) + P(1)$$

$$= 0.2 + 0.4$$

$$= 0.6$$