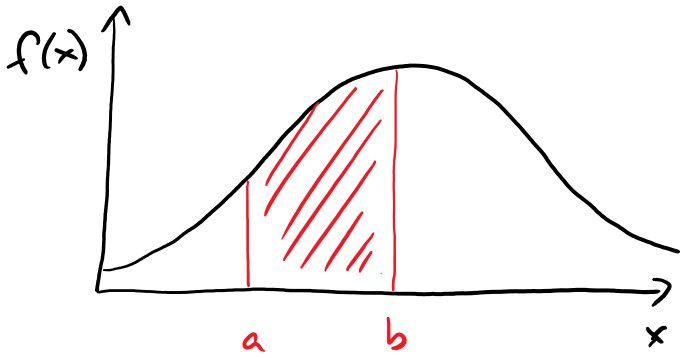
Section 5: cont'd

Monday, March 27, 2017 1:01 PM

if X is a continuous random variable the probability can be calculated as the area under a curve



the probability that X is between a and b is the area under this curve from a to b

the curve is called the probability density function (pdf)

note: the total area under the curve is equal to 1 (one) (or 100%)

$P(x = a) = 0$ because there are an infinite number of values a can have

which means

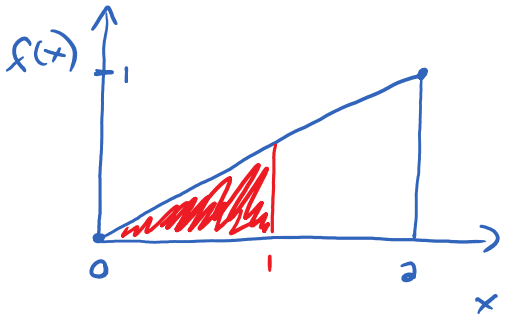
$$P(x > a) = P(x \geq a)$$

example: Let x be the amount of time for which a book is checked out of the library if it is on 2-hour reserve

suppose that

$$f(x) = \begin{cases} 0.5x & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Calculate the probability that $x \le 1$ hour



$P(x < 1)$ = shaded area of graph

$= \frac{1}{2} bh$

$= \frac{1}{2}(1)(\frac{1}{2})$

$= \frac{1}{4}$ or 0.25 or 25%

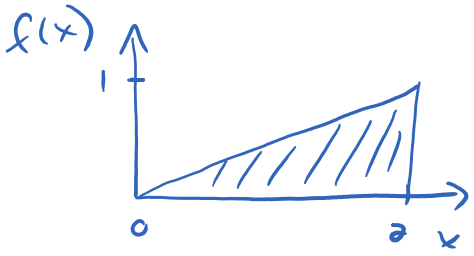
or if you insist

$$P(0 < x < 1) = \int_0^1 f(x)\,dx$$

$$= \int_0^1 \frac{1}{2} x\,dx$$

$$= \frac{1}{4} x^2 \Big|_0^1 = \frac{1}{4}$$

b) verify that the area under the curve is equal to one.



total area = $\frac{1}{2}bh$
$= \frac{1}{2}(2)(1)$
$= 1$

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how do you find the mean and standard deviation for a continuous distribution?

mean:

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

variance:

the annoying formal definition:

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

DON'T USE THIS!

the useful formula:

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

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going back to the book question, calculate the mean time a book is checked out and also the std dev.

recall $f(x) =$ [illegible]

recall $f(x) = \begin{cases} 0.5x & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{0}^{2} x\,(0.5x)\, dx$$

$$= \int_{0}^{2} \frac{1}{2} x^2\, dx$$

$$= \frac{x^3}{6}\Big|_0^2 = \frac{8}{6} = \frac{4}{3} \text{ or } 1.\overline{3} \text{ hours}$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$

$$= \int_{0}^{2} x^2 (0.5x)\, dx - \mu^2$$

$$= \int_{0}^{2} \frac{1}{2} x^3\, dx - \left(\frac{4}{3}\right)^2$$

$$= \frac{1}{8} x^4 \Big|_0^2 - \frac{16}{9}$$

$$= 2 - \frac{16}{9}$$

$$= \frac{2}{9}$$

$$\sigma = \sqrt{\sigma^2} = \frac{\sqrt{2}}{3} \approx 0.47 \text{ hours}$$

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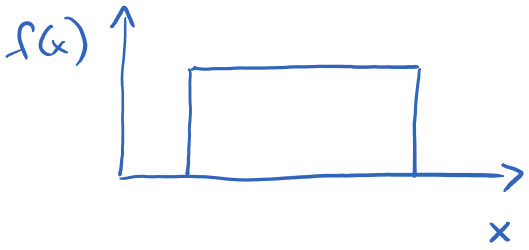
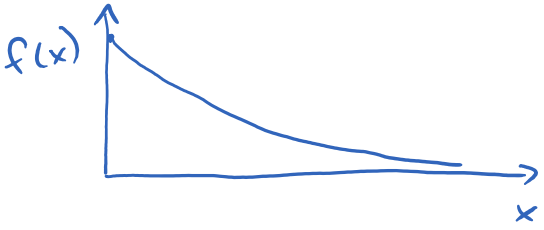
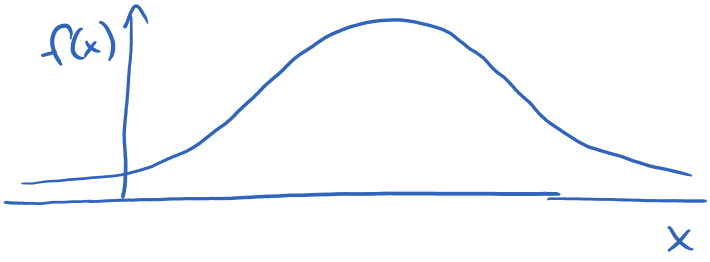
examples of continuous distributions:



uniform distribution



exponential distribution



normal distribution
(Bell curve)