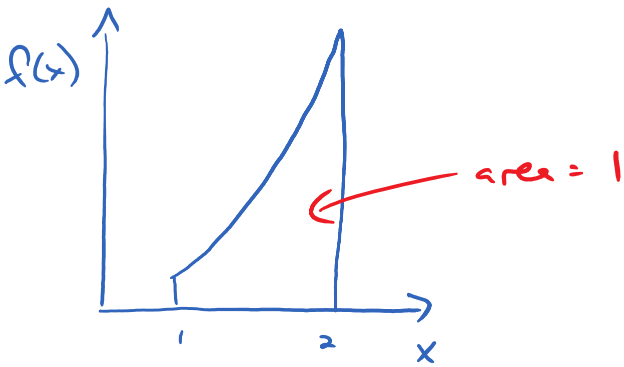
Section 5: cont'd

Tuesday, March 28, 2017 2:00 PM

from lecture handout:

(2) find the value of k that makes this a valid pdf:

$$f(x) = \begin{cases} kx^7 & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$



$$1 = \int_{-\infty}^{\infty} f(x)\,dx$$

$$1 = \int_1^2 kx^7\,dx$$

$$1 = \frac{k}{8}x^8 \Big|_1^2$$

$$= \frac{k}{8}(2^8 - 1^8)$$

$$= \frac{k}{8}(255)$$

$$k = \frac{8}{255}$$

(3) $f(x) = \Big\{ \frac{1}{\quad}$ for $0 \le x \le 11$

③ $f(x) = \begin{cases} \dfrac{1}{\ln 12\,(x+1)} & \text{for } 0 \le x \le 11 \\ 0 & \text{otherwise} \end{cases}$

a) probability that x is exactly 3 hours
b) " " " " between 2 and 4 hours

a) $P(x = 3) = 0$

b) $$P(2 < x < 4) = \int_a^b f(x)\,dx$$

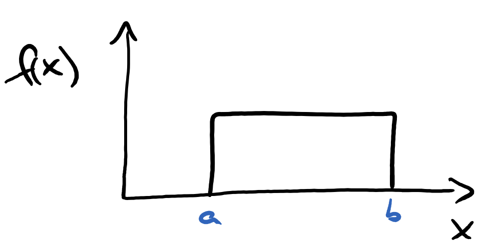
$$= \int_2^4 \frac{1}{\ln 12\,(x+1)}\,dx$$

$$= \frac{1}{\ln 12} \ln(x+1) \Big|_2^4$$

$$= \frac{1}{\ln 12} (\ln 5 - \ln 3)$$

$$\approx 0.21$$

Continuous uniform probability distribution



some constant

a b x

some

$$f(x) = \begin{cases} k & \text{for } a \le x \le b \\ 0 & \end{cases}$$

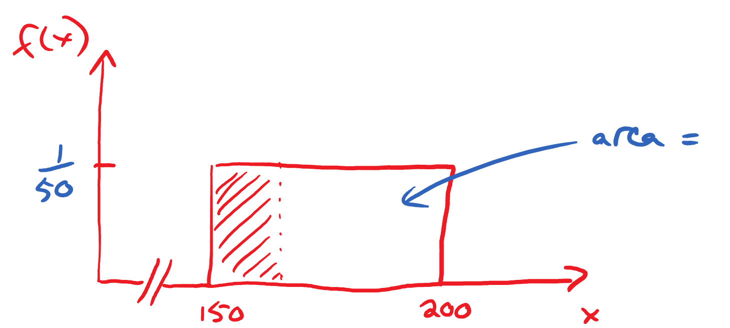
example: suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a uniform random variable with values between 150 and 200 mm. Any sheets less than 160 cm thick must be scrapped because they are unacceptable to buyers.

a) Calculate the fraction of steel sheets produced by this machine that must be scrapped.

b) Calculate the mean and std dev of the thickness of sheets produced.

c) What is the probability that a randomly selected sheet will lie within one std dev of the mean?

a)



area = $l \cdot w$

$1 = 50k$

$P(x < 160) = 1/5$ (area between 150 & 160)

or 20%

b) $\mu = 175$ mm (because of symmetry)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

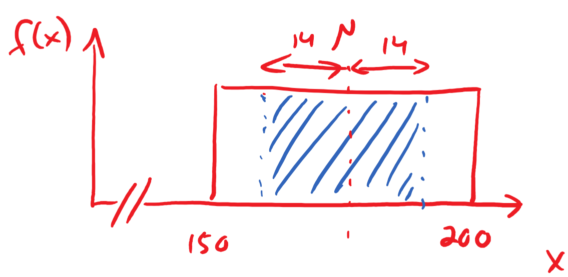
$$= \int_{150}^{200} x^2 \frac{1}{50}\,dx - 175^2$$

$$= \left.\frac{x^3}{150}\right|_{150}^{200} - 175^2$$

$$= 208.\overline{3}$$

$$\sigma = \sqrt{\sigma^2} \approx 14.4 \text{ mm}$$
$$\approx 14 \text{ mm}$$

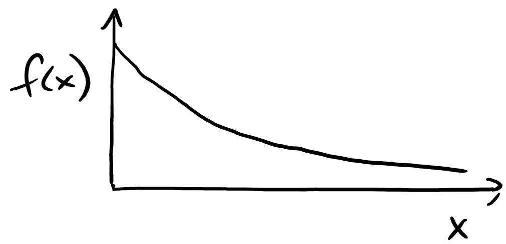
c)



$$P(161 < x < 189) = 28 \cdot \frac{1}{50}$$

$$= 56\%$$

the exponential distribution



$$f(x) = \begin{cases} k e^{-kx} & \text{for } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

note: $k > 0$

example: suppose that a system containing a certain type of battery has a "time to failure" of that component which is modeled nicely by an exponential distribution with $k = 1/5$. Assume the unit for time is years.

What is the probability that a given component will last less than 8 years? more than 8 years?

$$f(x) = \begin{cases} \frac{1}{5} e^{-1/5 x} & \text{for } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$P(x < 8) = \int_0^8 \frac{1}{5} e^{-x/5} \, dx$$

$$P(x<8) = \int_0^{8} \frac{1}{5} e^{-x/5}\, dx$$

$$= \frac{1}{5} \frac{e^{-x/5}}{-1/5} \Big|_0^8$$

$$= -e^{-x/5} \Big|_0^8$$

$$= -e^{-8/5} + e^{0}$$

$$= 1 - e^{-8/5}$$

$$\approx 0.798103$$

$$\approx 79.8\%$$

so $P(x>8) = 100\% - 79.8\% = 20.2\%$