Section 7: cont'd

Monday, April 3, 2017 1:00 PM

example: consider the population 1, 2, 4, 7

it has mean $\mu = 3.5$ and std dev $\sigma = 2.29$

now take samples of size 2

possible samples	$\bar{x}$
1, 2	1.5
1, 4	2.5
1, 7	4
2, 4	3
2, 7	4.5
4, 7	5.5

what happens when we take the average of these values?

mean $\bar{\bar{x}} = 3.5$

but std dev $\sigma_{\bar{x}} = 1.32$

the spread of the averages is smaller than the spread of the individual values

Conclusion: $\bar{x}$ values (averages/means) are more tightly clustered than the original population

Central Limit Theorem:

When the sample size ≥ 30, the set of $\bar{x}$ values is approximately normal with

mean μ

std dev $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Can you use the standard normal table for $\bar{x}$?

group size	original variable is normal	original variable is not normal
<30	✓	✗
≥ 30	✓	✓

so if $n \geq 30$, then can automatically use

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

in the standard normal distribution

note: $\sigma_{\bar{x}}$ is commonly called the standard error SE

so we write $SE = \dfrac{\sigma}{\sqrt{n}}$

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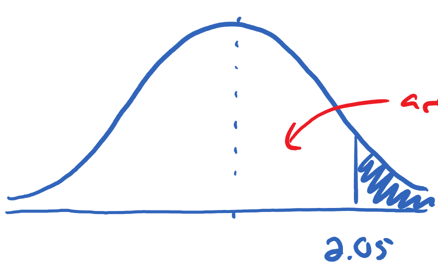
① large class has test average of 72 with a std dev of 8. Take a random sample of n tests. Find the prob that n tests average to more than 75 if

a) $n = 30$
b) $n = 80$

a) is $n \geq 30$? ✓ so $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$z = \frac{75 - 72}{8/\sqrt{30}}$$

$$= 2.05$$



area = 0.4798

$P(\bar{x} > 75) = 0.5 - 0.4798$
$= 0.0202$
or 2%

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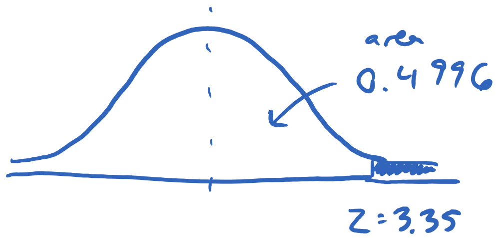
b) $n \geq 30$ ✓

$(n = 80)$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{75 - 72}{8/\sqrt{80}} = 3.35$$



$$P(\bar{x} > 75) = 0.5 - 0.4996$$
$$= 0.0004$$

$$\boxed{0.04\%}$$

② checked baggage

$\mu = 21$ kg
$\sigma = 4$ kg
$n = 40$ bags

find prob that average mass is

a) between 20 and 23 kg

$n \geq 30$ ✓

$(n = 40)$

$$z_{low} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{20 - 21}{4/\sqrt{40}}$$

$$= -1.58$$

$$Z_{high} = \frac{23-21}{4/\sqrt{40}}$$

$$= 3.16$$

area 0.4429

area 0.4992

$$P(20 < \bar{x} < 23) = 0.4429 + 0.4992$$

$$= 0.9421$$

so 94%