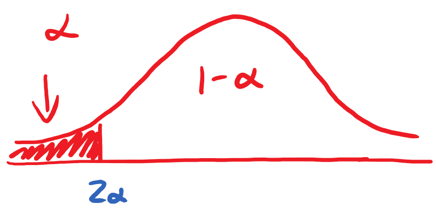
Section 8: cont'd

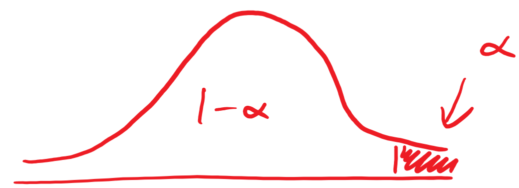
Thursday, April 6, 2017 1:36 PM

suppose we are looking at the concentration of pollutants in a harbour. Sometimes we are only interested in the upper limit of pollutants (if level of pollution is lower than expected, that's just fine!)

→ One-sided confidence interval



lower band



upper limit / upper band

$$\mu = \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$\mu = \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}$$

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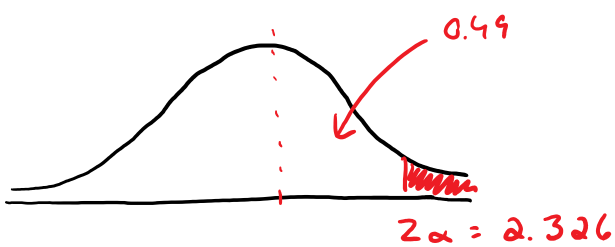
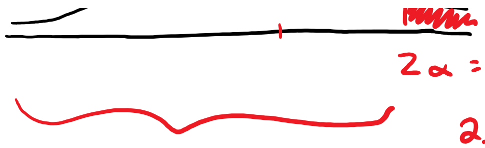
(6) 30 randomly selected water samples have a mean pollution concentration of 48.1 ppm with a standard deviation of 6.2 ppm. Find a 99% upper confidence band for the mean pollution concentration in the body of water.





$z_\alpha = 2.326$

0.99

2.33 also acceptable

$$\mu = \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$= 48.1 + 2.326\left(\frac{6.2}{\sqrt{30}}\right)$$

$$= 50.7$$

so with 99% confidence, the mean is less than (or equal to) 50.7 ppm

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for confidence intervals involving SMALL samples we cannot use the central limit theorem

we need a different table!

the t-distribution curve is flatter than the z-curve, with more area in the tails

the flatness varies depending on a variable called the "degrees of freedom"

$df = n - 1$ where n = sample size

assumption: random sample with size < 30, from a normally distributed population, with σ unknown (all you have is $\bar{x}$ and s)

$$\mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

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(8) The radius of a certain type of ball bearing is normally distributed. A random sample of 10 ball bearings has a mean radius of 4.9 cm with a std dev of 0.9 cm. Find a 95% confidence interval for the mean radius of all ball bearings of this type.

n $\bar{x}$ s $1-\alpha$

$n \geq 30$? ✗   t-table

$$\mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

df = 9

$1-\alpha = 95\%$, $\alpha = 5\%$

$\alpha/2 = 2.5\%$

$t_{0.025} = 2.262$

$$= 4.9 \pm 2.262\left(\frac{0.9}{\sqrt{10}}\right)$$

$$= 4.9 \pm 0.6$$

95% CI from 4.3 to 5.5 cm