Review:

Thursday, April 13, 2017 1:33 PM

A boat is being towed, when the tow-rope breaks. At the time that the rope breaks, the boat is traveling at 4 m/s. Ten seconds later, the boat is traveling at 1 m/s. Assume that the drag force on the boat is proportional to the square root of the boat's speed.

a) write an expression giving the relationship between the boat's acceleration and its speed.

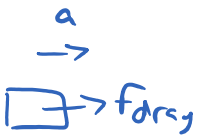
b) rewrite your answer to (a) as a DE

c) solve the DE.

a) $F_{drag} \propto \sqrt{v}$

$a \propto \sqrt{v}$

$a = k\sqrt{v}$



$ma = f_{drag}$

$ma = k\sqrt{v}$

$a = \left(\frac{k}{m}\right)\sqrt{v}$

c

b) $a = \frac{dv}{dt}$

$\frac{dv}{dt} = k\sqrt{v}$

c) $\int \frac{dv}{\sqrt{v}} = \int k\,dt$

$\frac{v^{1/2}}{1/2} = kt + C$

$2\sqrt{v} = kt + C$

at $t=0$, $v=4$
$t=10$, $v=1$

at $t=0$

$2\sqrt{4} = 0 + C$ so $C = 4$

at $t=10$, $2\sqrt{1} = 10k + 4$

$2 = 10k + 4$

$-2 = 10k$

$k = -1/5$

$2\sqrt{v} = -\frac{1}{5}t + 4$

$\sqrt{v} = -\frac{1}{10}t + 2$

$\int \frac{x^2 + x + 1}{x+2}\,dx$

← long division
degree num ≥ denom

$$\begin{array}{r} x - 1 \\ x+2 \overline{)\ x^2 + x + 1} \\ \underline{x^2 + 2x} \\ -x + 1 \end{array}$$

$$\frac{-x-2}{3}$$

$$\int \frac{x^2+x+1}{x+2}\,dx = \int \left[x - 1 + \frac{3}{x+2}\right] dx$$

$$= \frac{x^2}{2} - x + 3\ln|x+2| + C$$

$$\int x \sin x \, dx$$ ← product, so parts

D	I
x	$\sin x$
1	$-\cos x$
0	$-\sin x$

(+ then −)

LIATE

$$\int x \sin x \, dx = -x\cos x - (-\sin x) + C$$
$$= -x\cos x + \sin x + C$$

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method #2

$$\int x \sin x \, dx$$

$u = x$ $\quad dv = \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x \, dx = -x\cos x - \int -\cos x \, dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + C$$

$$\int \sqrt{x} \ln x \, dx$$

LIATE

D	I
$\ln x$	$x^{1/2}$
$\frac{1}{x}$	$\frac{2}{3}x^{3/2}$

(+ from $\ln x$ to $\frac{2}{3}x^{3/2}$; − from $\frac{1}{x}$ to $\frac{2}{3}x^{3/2}$)

$$\int \sqrt{x} \ln x \, dx = \frac{2}{3}x^{3/2}\ln x - \int \frac{1}{x}\left(\frac{2}{3}x^{3/2}\right) dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2} \, dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \left(\frac{2}{3}\right)^2 x^{3/2} + C$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

integration check list:

① can you substitute?

– want to get $\int e^u du$

↑ or $\sin u$, $\cos u$, $\tan u$, $\frac{1}{u}$, u^n

② is it $\int \frac{1}{1+u^2} du$ or $\int \frac{1}{\sqrt{1-u^2}} du$?

arctan arcsin

$\int \frac{e^x dx}{1+e^{2x}}$ ↑ with $u = e^x$

③ is it a product? $\int x \cos x \, dx$

parts!

④ can you use a trig identity?

$\int \frac{\sin 2x}{1-\sin^2 x} dx = \int \frac{2 \sin x \cos x}{1-\sin^2 x} dx$

⑤ can you factor the denominator?

$\int \frac{8 \cdot 2x}{x^2-4} dx$ $\Rightarrow$ partial fractions