Tutorial

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Integrate

$$\int \frac{6e^x\,dx}{\sqrt{3e^x+2}} = \int \frac{2\,du}{\sqrt{u}}$$

let $u = 3e^x+2$
$du = 3e^x\,dx$
$\frac{du}{3} = e^x\,dx$

$$= \int 2u^{-1/2}\,du$$

$$= 2\frac{u^{1/2}}{1/2} + C$$

$$= 4\sqrt{3e^x+2} + C$$

$$\boxed{\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad \text{for } n \neq -1}$$

$$\int \frac{4\sqrt{t}}{t\sqrt{t}+1}\,dt = \int \frac{4t^{1/2}}{t^{3/2}+1}\,dt$$

let $u = t^{3/2}+1$
$du = \frac{3}{2}t^{1/2}\,dt$
$\frac{2}{3}du = t^{1/2}\,dt$

$$= \int \frac{4\cdot\frac{2}{3}\,du}{u}$$

$$= \frac{8}{3}\ln|u| + C$$

$$= \frac{8}{3} \ln \left| t\sqrt{t} + 1 \right| + C$$

$$\int \frac{e^{2x}\,dx}{1+e^{2x}} = \int \frac{du}{2u}$$

let $u = 1 + e^{2x}$
$du = 2e^{2x}\,dx$
$\frac{1}{2}du = e^{2x}\,dx$

$$= \frac{1}{2} \ln |u| + C$$

$$= \frac{1}{2} \ln \left| 1 + e^{2x} \right| + C$$

$$\int \frac{e^{x}\,dx}{1+e^{2x}} = \int \frac{du}{1+u^2}$$

let $u = e^x$
$du = e^x\,dx$

$e^{2x} = e^x \cdot e^x$

$$= \tan^{-1} u + C$$

$$= \tan^{-1} e^x + C$$

$$\int \frac{7-x}{x^2+x-2}\,dx = \int \frac{7-x}{(x+2)(x-1)}\,dx$$

$$(x+2)(x-1)\left[\frac{7-x}{(x+2)(x-1)}\right] = \left[\frac{A}{x+2} + \frac{B}{x-1}\right](x+2)(x-1)$$

$$7 - x = A(x-1) + B(x+2)$$

let $x = 1$:

$$7 - 1 = A \cdot 0 + B(1+2)$$

$$6 = 3B$$
$$B = 2$$

let $x = -2$:

$$7+2 = A(-2-1) + B \cdot 0$$
$$9 = -3A$$
$$A = -3$$

$$\int \frac{7-x}{(x+2)(x-1)}\,dx = \int \left[\frac{-3}{x+2} + \frac{2}{x-1}\right] dx$$

$$= -3\ln|x+2| + 2\ln|x-1| + C$$

$$\int_0^1 \left[\int_{x^2}^{x} xy\,dy\right] dx = \int_0^1 \left[\frac{xy^2}{2}\Big|_{y=x^2}^{y=x}\right] dx$$

$$= \int_0^1 \left[\frac{x^3}{2} - \frac{x^5}{2}\right] dx$$

$$= \left(\frac{x^4}{8} - \frac{x^6}{12}\right)\Big|_0^1$$

$$= \frac{1}{8} - \frac{1}{12}$$

$$= \frac{3}{24} - \frac{2}{24} = \frac{1}{24}$$

Find all four second partial derivatives of:

$$f(x,y) = \cos xy$$

$$\frac{\partial f}{\partial x} = -\sin(xy) \cdot y = -y \sin xy$$

$$\frac{\partial f}{\partial y} = -\sin(xy) \cdot x = -x \sin xy$$

$$\frac{\partial^2 f}{\partial x^2} = -y^2 \cos xy$$

differentiating this wrt x

$$\frac{\partial^2 f}{\partial y^2} = -x^2 \cos xy$$

differentiate wrt y

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(-x \sin xy\right)$$

$$= -\sin xy + -xy \cos xy$$

$$\frac{\partial^2 f}{\partial y \partial x} = \text{same} =$$