# MATH 193 FINAL EXAM REVIEW QUESTIONS

1. Evaluate $\int_0^2 xe^{7x^2}\,dx$.
2. Evaluate $\int (x^2-3)e^{7x}\,dx$.
3. Evaluate $\int \frac{t}{\sqrt[3]{6t^2+1}}\,dt$.
4. Evaluate $\int \frac{6e^{3x}}{1+4e^{3x}}\,dx$.
5. Evaluate $\int \ln x\,dx$.
6. Evaluate $\int \frac{7x}{\sqrt{5-9x^4}}\,dx$.
7. Evaluate $\int \frac{\tan^{-1}x}{1+x^2}\,dx$.
8. Evaluate $\int_1^3 \frac{1}{x^2-25}\,dx$.
9. Let $f(x,y)=y\cos x+xe^y+5xy$. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y\partial x}$.
10. Find the first-octant volume below $z=x+y$ bounded by $y=2x$ and $y=x^2$.
11. Show that $y=x\ln x-Cx$ is a solution to $x+y-xy'=0$.
12. Find an explicit solution to $(x^3-2)^4dy+27x^2dx=0$ if the solution passes through the point $(2,5)$.
13. Find an explicit solution to the following DE:

$$(\cos x)\frac{dy}{dx}=2-y\sin x$$

14. An object with a mass of 1.00 kg slides down an inclined plane. The component of gravity down the plane is 4.00 N, and the motion is retarded by a force numerically equal to the velocity. If the object starts from rest, what is its velocity after 4.00 seconds? Start with an appropriate DE and show all your work.

15. Consider $y'' + 6y' + ky = 0$. Solve it for:

    (a) $k = 8$

    (b) $k = 9$

    (c) $k = 10$

16. The following DE's have complementary solution $y_c = C_1 + C_2 e^{5x}$. State the form of the particular solution $y_p$ for each DE.

    (a) $y'' - 5y' = 4e^{5x} + e^{6x}$

    (b) $y'' - 5y' = 2x \sin x$

    (c) $y'' - 5y' = 1 - e^{-2x}$

17. Solve $\dfrac{d^2y}{dx^2} + 4y = 2\sin x$ given $y(\pi) = 14$ and $y'(\pi) = -8$.

18. A 3 kg mass is attached to a spring with spring constant 24 N/m. There is a damping force equal to 18 times the velocity, as well as an external force $f(t) = t$. The mass is initially 50 cm above the equilibrium position with a downwards velocity of 10 cm/s. Find the equation of motion. Start with an appropriate DE and show all your work.

19. Consider the following population of times to complete a lab report (in minutes):

$$68, 72, 55, 41$$

    (a) Compute the mean.

    (b) Compute the median.

    (c) Compute the standard deviation.

    (d) If the smallest measurement in the population is increased slightly, what can you say about the new variance?

20. A shipment contains two parts: A and B. The probability that part A is defective is 2.3%; the probability that part B is defective is 1.9%. The probability that neither part is defective is 97.5%. Find the probability that both parts are defective.

21. Consider the random variable below:

| $x$ | $P(x)$ |
|---|---|
| $-2$ | 0.1 |
| 3 | 0.6 |
| 7 | 0.3 |

    (a) Find $P(X \leq 4)$.

    (b) Find the expected value of $X$.

    (c) Find the variance of $X$.

(d) Find the standard deviation of $X$.

(e) Find the probability that a value of $X$ lies within 1.5 standard deviations of the mean.

22. If the probability is 0.05 that a certain wide-flange column will fail under a given axial load, what is the probability that among 16 such columns, at most two will fail?

23. A hotline receives an average of 1.2 calls per minute. What is the probability that the hotline receives at least 4 calls in the next three minutes?

24. Find the mean and standard deviation of $X$ with probability density function:

$$f(x) = \begin{cases} \frac{x^2}{4} & 1 \leq x \leq 2 \\ \frac{5x}{72} & 2 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

25. $X$ is a normal random variable with a mean of 150 and a SD of 35. Find the cut-off for the largest 7% of measurements.

26. The heights of adult males in a certain city have a mean of 67 inches and a standard deviation of 4 inches. What is the probability that 45 randomly-selected adult males have a mean height between 65.5 and 66 inches?

27. At a paper factory, the paper length is known to have a standard deviation of 0.08 inches. In a random sample of 80 sheets, the mean length is found to be 11.03 inches. Consider the mean length among all sheets of paper produced at the factory. Find a:

(a) 98% confidence interval for the mean.

(b) 90% upper confidence bound for the mean.

(c) 95% lower confidence bound for the mean.

28. At Crunchy Cereal Company, the mass of cereal boxes has a standard deviation of 5.6 grams. We want a 99% confidence interval for the mean mass among all cereal boxes with a margin of error less than 1.5 grams. Find the minimum sample size.

29. Fifteen random water samples taken from the inner harbour yield a mean nitrate ion concentration of 37.2 ppm with a standard deviation of 4.6 ppm. The ion concentrations are normally distributed throughout the inner harbour. Consider the mean ion concentration in the inner harbour. Find a:

(a) 99% confidence interval for the mean.

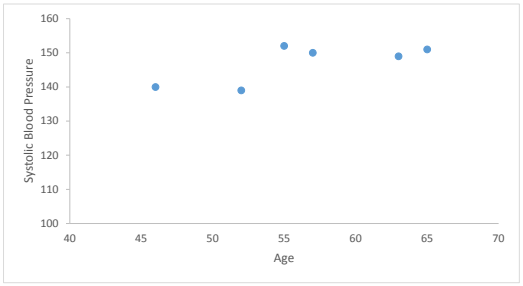
(b) 90% upper confidence bound for the mean.

(c) 95% lower confidence bound for the mean.

30. Consider the blood pressure data below, where

$$x = \text{age, and}$$
$$y = \text{systolic blood pressure}$$

| $x$ | $y$ |
|---|---|
| 46 | 140 |
| 65 | 151 |
| 63 | 149 |
| 57 | 150 |
| 52 | 139 |
| 55 | 152 |




The equation of the best-fit line is $\hat{y} = 0.61x + 112.59$ and the coefficient of determination is 0.5477.

(a) Find the correlation coefficient.

(b) What percentage of the variation in $y$ is accounted for by the best-fit line?

(c) What systolic blood pressure is predicted for a 50-year-old?

(d) Why should we not use $\hat{y}$ to predict the systolic blood pressure for a 70-year-old?

(e) What age corresponds to a systolic blood pressure of 145?

## Answers

1. $\frac{1}{14}(e^{28}-1)$
2. $\frac{1}{7}e^{7x}\left(x^2-\frac{2}{7}x-\frac{145}{49}\right)+C$
3. $\frac{1}{8}(6t^2+1)^{2/3}+C$
4. $\frac{1}{2}\ln|1+4e^{3x}|+C$
5. $x\ln x-x+C$
6. $\frac{7}{6}\sin^{-1}\left(\frac{3x^2}{\sqrt{5}}\right)+C$
7. $\frac{1}{2}(\tan^{-1}x)^2+C$
8. $-\frac{1}{10}\ln\frac{8}{3}=\frac{1}{10}\ln\frac{3}{8}$
9. $\frac{\partial^2 f}{\partial x^2}=-y\cos x$
   $\frac{\partial^2 f}{\partial y\partial x}=-\sin x+e^y+5$
10. $\frac{52}{15}$
11. If $y=x\ln x-Cx$ then $y'=1+\ln x-C$, so
$$\begin{aligned} x+y-xy' &= 0 \\ x+(x\ln x-Cx)-x(1+\ln x-C) &= 0 \\ &\vdots \\ 0 &= 0 \end{aligned}$$
12. $y=\frac{3}{(x^3-2)^3}+\frac{359}{72}$
13. $y=2\sin x+C\cos x$
14. 3.93 m/s
15. (a) $y=C_1e^{-4x}+C_2e^{-2x}$
    (b) $y=(C_1+C_2x)e^{-3x}$
    (c) $y=e^{-3x}(C_1\cos x+C_2\sin x)$
16. (a) $y_p=Axe^{5x}+Be^{6x}$
    (b) $y_p=Ax\sin x+Bx\cos x+C\sin x+D\cos x$
    (c) $y_p=Ax+Be^{-2x}$
17. $y=14\cos 2x-\frac{11}{3}\sin 2x+\frac{2}{3}\sin x$
18. $x=\frac{211}{480}e^{-4t}-\frac{109}{120}e^{-2t}+\frac{1}{24}t-\frac{1}{32}$

19. (a) 59
    (b) 61.5
    (c) 12.1
    (d) It will decrease.

20. 0.017

21. (a) 0.7
    (b) 3.7
    (c) 6.81
    (d) 2.6
    (e) 0.9

22. 0.96

23. 0.48

24. $\mu = \frac{965}{432} \approx 2.23$, $\sigma \approx 0.85$

25. 202

26. 0.0406

27. (a) $11.01 < \mu < 11.05$ inches
    (b) $\mu < 11.04$ inches
    (c) $\mu > 11.02$ inches

28. 93

29. (a) $33.7 < \mu < 40.7$ ppm
    (b) $\mu < 38.8$ ppm
    (c) $\mu > 35.1$ ppm

30. (a) 0.74
    (b) 54.77%
    (c) 143
    (d) $x = 70$ is outside the dataset range $46 \leq x \leq 65$
    (e) 53