Math 193 Final Exam Review SOLUTIONS #22-30

22. this is the BINOMIAL set-up

with X = number of columns that fail, $n = 16$, $p = 0.05$

$$P(X=x) = {}_nC_x \cdot p^x q^{n-x}$$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}_{16}C_0 \cdot (0.05)^0 (0.95)^{16} + {}_{16}C_1 \cdot (0.05)^1 (0.95)^{15} + {}_{16}C_2 \cdot (0.05)^2 (0.95)^{14}$$

$$= 0.96$$

23. this is the POISSON set-up

with X = number of calls in 3 minutes

$$\lambda = 1.2 \frac{\text{calls}}{\text{min}} = 3.6 \frac{\text{calls}}{3 \text{ min}}$$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(X \ge 4) = 1 - P(X<4)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[\frac{3.6^0 e^{-3.6}}{0!} + \frac{3.6^1 e^{-3.6}}{1!} + \frac{3.6^2 e^{-3.6}}{2!} + \frac{3.6^3 e^{-3.6}}{3!}\right]$$

$$= 0.48$$

24. $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_1^2 x \cdot \frac{x^2}{4}\,dx + \int_2^4 x \cdot \frac{5x}{72}\,dx$$

$$= \frac{1}{4}\int_1^2 x^3\,dx + \frac{5}{72}\int_2^4 x^2\,dx$$

$$= \frac{1}{16} x^4 \Big|_1^2 + \frac{5}{216} x^3 \Big|_2^4$$

$$= \frac{1}{16}\left(2^4 - 1^4\right) + \frac{5}{216}\left(4^3 - 2^3\right)$$

$$= \frac{965}{432} = 2.23$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_1^2 x^2 \cdot \frac{x^2}{4}\,dx + \int_2^4 x^2 \cdot \frac{5x}{72}\,dx$$

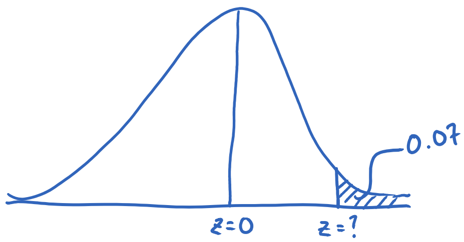
$$= \frac{1}{4}\int_1^2 x^4\,dx + \frac{5}{72}\int_2^4 x^3\,dx$$

$$= \frac{1}{20} x^5 \Big|_1^2 + \frac{5}{288} x^4 \Big|_2^4$$

$$= \frac{1}{20}\left(2^5 - 1^5\right) + \frac{5}{288}\left(4^4 - 2^4\right) = \frac{343}{60}$$

$\sigma^2 = E(X^2) - \mu^2 = \frac{343}{60} - \left(\frac{965}{432}\right)^2 = 0.73$ so $\sigma = \sqrt{0.73} \approx 0.85$

25. $\mu = 150$, $\sigma = 35$



reverse look-up:
area = $0.5 - 0.07 = 0.43 \Rightarrow z = 1.48$

$$z = \frac{x - \mu}{\sigma}$$

$$1.48 = \frac{x - 150}{35}$$

$$x = 1.48(35) + 150 = 201.8$$

26. $\mu = 67$, $\sigma = 4$, $n = 45$

we want $P(65.5 \leq \bar{x} \leq 66)$

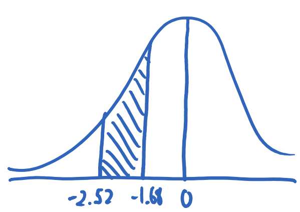
large sample means are NORMAL

$$\bar{x} = 65.5 \Rightarrow z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{65.5 - 67}{4/\sqrt{45}} = -2.52$$

$$\bar{x} = 66 \Rightarrow z = \frac{66 - 67}{4/\sqrt{45}} = -1.68$$

$$P(65.5 \leq \bar{x} \leq 66) = P(-2.52 \leq z \leq -1.68)$$

=



$= 0.4941 - 0.4535 = 0.0406$

27. $\sigma = 0.08$, $n = 80$, $\bar{x} = 11.03$

a) $C = 0.98$

$CI \Rightarrow z_{\alpha/2} = 2.326$

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 11.03 \pm 2.326\left(\frac{0.08}{\sqrt{80}}\right) = 11.03 \pm 0.02$$

$11.01 < \mu < 11.05$ inches

b) $C = 0.9$

$UCB \Rightarrow z_{\alpha} = 1.282$

$$\bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} = 11.03 + 1.282\left(\frac{0.08}{\sqrt{80}}\right) = 11.04$$

$\mu < 11.04$ inches

c) $C = 0.95$

$LCB \Rightarrow z_{\alpha} = 1.645$

$$\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}} = 11.03 - 1.645\left(\frac{0.08}{\sqrt{80}}\right) = 11.02$$

$\mu > 11.02$ inches

28. $\sigma = 5.6$, $C = 0.99$ $\Rightarrow$ $z_{\alpha/2} = 2.576$

$$ME < 1.5$$
$$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < 1.5$$
$$2.576\left(\frac{5.6}{\sqrt{n}}\right) < 1.5$$
$$14.4256 < 1.5\sqrt{n}$$
$$\sqrt{n} > 9.617$$
$$n > 92.5$$

so $n = 93$ is the minimum sample size

29. $n = 15$, $\bar{x} = 37.2$, $s = 4.6$

a) $C = 0.99$

$$CI \Rightarrow \left.\begin{array}{l} \alpha = 0.01 \Rightarrow \alpha/2 = 0.005 \\ df = n-1 = 15-1 = 14 \end{array}\right\} t_{\alpha/2} = 2.977$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 37.2 \pm 2.977\left(\frac{4.6}{\sqrt{15}}\right) = 37.2 \pm 3.5$$

$$33.7 < \mu < 40.7 \text{ ppm}$$

29. b) $C = 0.9$

$UCB \Rightarrow \left.\begin{array}{l}\alpha = 0.1\\ df = 14\end{array}\right\} t_\alpha = 1.345$

$$\bar{x} + t_\alpha \frac{s}{\sqrt{n}} = 37.2 + 1.345\left(\frac{4.6}{\sqrt{15}}\right) = 38.8$$

$\mu < 38.8$ ppm

c) $C = 0.95$

$LCB \Rightarrow \left.\begin{array}{l}\alpha = 0.05\\ df = 14\end{array}\right\} t_\alpha = 1.761$

$$\bar{x} - t_\alpha \frac{s}{\sqrt{n}} = 37.2 - 1.761\left(\frac{4.6}{\sqrt{15}}\right) = 35.1$$

$\mu > 35.1$ ppm

30. a) $r^2 = 0.5477$
$r = 0.7401$

b) 54.77%

c) $x = 50 \Rightarrow \hat{y} = 0.61(50) + 112.59 = 143.09$

d) $x = 70$ is outside the dataset range $46 \leq x \leq 65$

e) $\hat{y} = 145 \Rightarrow 145 = 0.61x + 112.59 \Rightarrow x \approx 53$