

MATH 193 PRACTICE QUESTIONS

1. Evaluate $\int_0^1 \frac{e^{9x}}{2 + 4e^{9x}} dx$.
2. Evaluate $\int_0^{\pi/4} \sqrt[4]{\tan x} \sec^2 x dx$.
3. Evaluate $\int \frac{x}{\sqrt{4 - x^2}} dx$.
4. Evaluate $\int \frac{4}{\sec x e^{\sin x}} dx$.
5. Evaluate $\int \frac{x}{\sqrt{1 + x}} dx$.
6. Evaluate $\int \frac{e^{-x}}{1 + e^{-2x}} dx$.
7. Evaluate $\int_0^{\pi/4} \frac{5 - 8 \sin(2x)}{\cos^2 x} dx$.
8. Evaluate $\int \frac{\ln x}{x^2} dx$.
9. Evaluate $\int \frac{3}{x^2 - 25} dx$.
10. Evaluate $\int \frac{5}{x^2 + 8x + 17} dx$.
11. Let $f(x, y) = e^x \cos y + e^{-2x} \tan y$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial^2 f}{\partial x \partial y}$.
12. Evaluate $\int_1^2 \int_x^{x^2} x^2 y dy dx$.
13. Find the first-octant volume below the surface $z = xy$ and inside the cylinder $x^2 + y^2 = 1$.
14. Show that $y = \sqrt{Cx - x^2}$ solves the following DE: $2xyy' + x^2 = y^2$.
15. Find an implicit solution to the following DE if $y = 1$ when $x = \pi$:

$$\sin x dy = x^3 \sin^2 x dx + \frac{3y \sin x}{x} dx$$

16. Solve the following DE explicitly for y :

$$6xydx + (x^2 + 7)dy = 0$$

17. According to Newton's law of cooling, the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium.

A cup of coffee is sitting on the counter in a 20°C room. The coffee's temperature is initially 90°C , and takes 5 minutes to reach 80°C . What is the coffee's temperature after 15 minutes? Show all your work, starting with an appropriate differential equation.

18. Consider $y'' + 8y' + ky = 0$. Solve it for:

- (a) $k = 7$.
- (b) $k = 16$.
- (c) $k = 18$.

19. The following DE's have complementary solution $y_c = C_1 \cos x + C_2 \sin x$. State the form of the particular solution y_p for each DE.

- (a) $y'' + y = 9x^2$
- (b) $y'' + y = 5 \cos x$
- (c) $y'' + y = e^{7x}$
- (d) $y'' + y = x^2 e^{4x}$

20. Solve $y'' + 7y' + 12y = \sin 2x$ given $y(0) = 4$ and $y'(0) = -3$.

21. A 2 kg mass is attached to a spring with spring constant 12 N/m. There is a damping force equal to 10 times the velocity, as well as an external force $f(t) = \sin 3t$. The mass is initially 20 cm below the equilibrium position with an upwards velocity of 30 cm/s. Find the equation of motion. Start with an appropriate DE and show all your work.

22. Consider the following sample of temperature readings (in $^\circ\text{C}$):

$$7, 4, -4, -2, 3, 1$$

- (a) Compute the mean.
 - (b) Compute the median.
 - (c) Compute the standard deviation.
 - (d) If we added 1.5 to every measurement in the sample, what would the new variance be?
23. In a class of 45 students, 8 live alone and 31 have a part-time job. Of those who have a part-time job, 3 live alone. What is the probability that a student doesn't live alone and doesn't have a part-time job?

24. A network password consists of 8 symbols chosen from the symbol set a, \dots, z, A, \dots, Z . Find the probability that a password:
- ends with yZ .
 - doesn't start with b .
 - starts with C or ends with C .
25. Project A has a 65% chance of earning \$8,000, a 15% chance of earning \$3,000, and a 20% chance of earning -\$5,000 (that is, losing \$5,000).
- Find the expected earnings of Project A.
 - Find the standard deviation of Project A's earnings.
 - Consider Project B. The standard deviation of Project B's earnings is \$6,200. Which project's earnings are more uncertain, Project A or Project B?
26. A switchboard receives an average of 1.8 calls per minute. Find the probability that the switchboard receives at least 3 calls in the next two minutes.
27. A multiple-choice quiz has 3 questions, each of which has 4 possible answers. A student guesses randomly on each question. Find the probability distribution for the number of questions the student gets right.
28. The time to complete an assignment (in hours) has probability density function

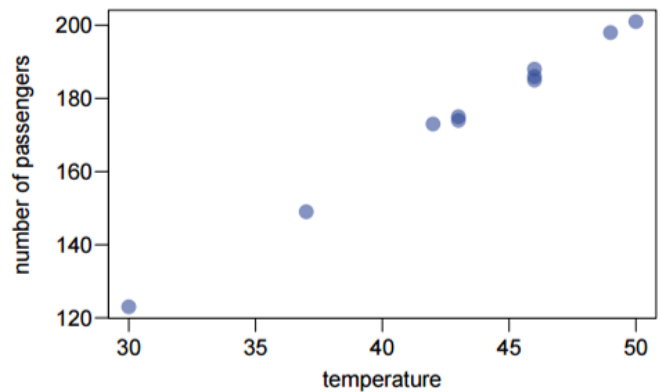
$$f(x) = \begin{cases} \frac{x^2}{20} & 0 \leq x \leq 3 \\ \frac{33(x+x^2)}{2440} & 3 < x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that an assignment takes:

- exactly 2.5 hours.
 - between 2.5 and 4.5 hours.
 - more than 4.5 hours.
29. In a certain town, the weights of adult males are normally distributed with a mean of 168 pounds. 30.23% of adult males in the town weigh between 148 and 168 pounds. Find the standard deviation of the weights.
30. A large class writes a test. The average test mark is 71 with a standard deviation of 6. Find the probability that 35 randomly-selected tests have an average mark of less than 68 or more than 73.

31. You are interested in the average emergency room (ER) wait time at your local hospital. You take a random sample of 50 patients who visited the ER over the past week. From this sample, the mean wait time was 30 minutes and the standard deviation was 20 minutes.
- Find a 95% confidence interval for the average ER wait time for the hospital.
 - Find the margin of error for the 95% confidence interval.
 - Find the margin of error for the 95% confidence interval if the standard deviation of the sample was 10 minutes instead of 20 minutes.
32. A city's transportation department is interested in studying the relationship between the temperature and the number of passengers that ride the main bus line in order to better serve their customers. For each of 10 hours, the manager recorded the temperature at the beginning of the hour, and the bus driver recorded the number of passengers that boarded the bus throughout that hour. Their findings are listed below.

temperature ($^{\circ}\text{F}$)	passengers
42	173
37	149
46	185
30	123
50	201
43	174
43	175
46	188
46	186
49	198



- Find the correlation coefficient r .
- Find the least squares regression equation, i.e. the best-fit line.
- Interpret the slope.
- Predict the number of passengers when the temperature is 45°F .
- For the first data point (42°F , 173 passengers), what is the residual?
- Can you predict the number of passengers when the temperature is at 65°F ?

Answers

1. $\frac{1}{36} (\ln |2 + 4e^9| - \ln 6)$ OR $\frac{1}{36} \ln \left(\frac{1+2e^9}{3} \right)$
2. $\frac{4}{5}$
3. $-\sqrt{4-x^2} + C$
4. $\frac{-4}{e^{\sin x}} + C$
5. $2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2} + C$
6. $-\tan^{-1}(e^{-x}) + C$
7. $5 - 16 \ln \sqrt{2}$ OR $5 - 8 \ln 2$
8. $\frac{-\ln x}{x} - \frac{1}{x} + C$
9. $\frac{3}{10} \ln |x-5| - \frac{3}{10} \ln |x+5| + C$ OR $\frac{3}{10} \ln \left| \frac{x-5}{x+5} \right| + C$
10. $5 \tan^{-1}(x+4) + C$
11. $\frac{\partial f}{\partial x} = e^x \cos y - 2e^{-2x} \tan y$
 $\frac{\partial f}{\partial y} = -e^x \sin y + e^{-2x} \sec^2 y$
 $\frac{\partial^2 f}{\partial x \partial y} = -e^x \sin y - 2e^{-2x} \sec^2 y$
12. $\frac{209}{35}$
13. $\frac{1}{8}$
14. If $y = \sqrt{Cx - x^2}$ then $y' = \frac{C - 2x}{2\sqrt{Cx - x^2}}$. So

$$2xyy' + x^2 = y^2$$

$$2x \cdot \sqrt{Cx - x^2} \cdot \frac{C - 2x}{2\sqrt{Cx - x^2}} + x^2 = (\sqrt{Cx - x^2})^2$$

$$\vdots$$

$$Cx - x^2 = Cx - x^2$$
15. $x^{-3}y = -\cos x + \frac{1}{\pi^3} - 1$
16. $y = \frac{k}{(x^2+7)^3}$
17. 64°C
18. (a) $y = C_1 e^{-7x} + C_2 e^{-x}$
 (b) $y = (C_1 + C_2 x) e^{-4x}$
 (c) $y = e^{-4x} (C_1 \cos \sqrt{2}x + C_2 \sin \sqrt{2}x)$
19. (a) $y_p = Ax^2 + Bx + C$

- (b) $y_p = Ax \cos x + Bx \sin x$
 (c) $y_p = Ae^{7x}$
 (d) $y_p = Ax^2e^{4x} + Bxe^{4x} + Ce^{4x}$
20. $y = \frac{-91}{10}e^{-4x} + \frac{171}{13}e^{-3x} + \frac{2}{65} \sin 2x - \frac{7}{130} \cos 2x$
21. $x = \frac{27}{65}e^{-2t} - \frac{11}{60}e^{-3t} - \frac{1}{156} \sin 3t - \frac{5}{156} \cos 3t$
22. (a) 1.5
 (b) 2
 (c) 4.0
 (d) no change
23. $\frac{9}{45} = 0.2$
24. (a) $\frac{1}{2704}$
 (b) $\frac{51}{52}$
 (c) $\frac{103}{2704}$
25. (a) \$4650
 (b) \$5131
 (c) Project B
26. 0.70
27.

x	$P(x)$
0	27/64
1	27/64
2	9/64
3	1/64
28. (a) 0
 (b) 0.55
 (c) 0.18
29. 23.5 pounds
30. 0.0259
31. (a) $24.5 < \mu < 35.55$ minutes
 (b) 5.5 minutes
 (c) 2.8 minutes
32. (a) 0.9984
 (b) $\hat{y} = 4.413 + 3.953x$
 (c) As the temperature increases by 1°F, the number of passengers increases by approx. 4.
 (d) 182
 (e) 3
 (f) no