Practice Questions #24-32

24. $2(26) = 52$ symbols

a) $P(\text{ends } y2) = \dfrac{n(\text{ends } y2)}{n(S)} = \dfrac{52^6 \cdot 1 \cdot 1}{52^8} = \dfrac{1}{52^2} = \dfrac{1}{2704}$

b) $P(\text{doesn't start } b) = \dfrac{51 \cdot 52^7}{52^8} = \dfrac{51}{52}$

c) $P(\text{starts } C \text{ or ends } C) = P(\text{starts } C) + P(\text{ends } C) - P(\text{starts and ends } C)$

$$= \frac{1 \cdot 52^7}{52^8} + \frac{52^7 \cdot 1}{52^8} - \frac{1 \cdot 52^6 \cdot 1}{52^8}$$

$$= \frac{103}{2704}$$

25. X = earnings of Project A (\$)

x	$P(x)$
8000	0.65
3000	0.15
-5000	0.2

a) $E(X) = \sum x P(x) = 8000(0.65) + 3000(0.15) + (-5000)(0.2)$
$= \$4650$

b) $\sigma^2 = E(X^2) - [E(X)]^2$

$E(X^2) = \sum x^2 P(x) = 8000^2(0.65) + 3000^2(0.15) + (-5000)^2(0.2)$
$= 47{,}950{,}000$

$\sigma^2 = 47950000 - 4650^2$
$= 26327500$

$\sigma = \sqrt{26327500} \approx \$5131$

c) $\sigma_A = 5131 < 6200 = \sigma_B$

so Project B's earnings are more uncertain

26. this is the set-up for the Poisson distribution with

X = number of calls in 2 minutes and

$\lambda = \frac{1.8 \text{ calls}}{\text{minute}} = \frac{3.6 \text{ calls}}{2 \text{ minutes}}$

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \frac{3.6^0 e^{-3.6}}{0!} - \frac{3.6^1 e^{-3.6}}{1!} - \frac{3.6^2 e^{-3.6}}{2!}$

$= 0.70$

27. this is the set-up for the binomial distribution with

X = number of correct answers
$n = 3$
$p = \frac{1}{4}$
$q = 1 - p = \frac{3}{4}$

$$P(X=0) = 3C0 \cdot \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$P(X=1) = 3C1 \cdot \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^2 = \frac{27}{64}$$

$$P(X=2) = 3C2 \cdot \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1 = \frac{9}{64}$$

$$P(X=3) = 3C3 \cdot \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 = \frac{1}{64}$$

x	$P(x)$
0	$27/64$
1	$27/64$
2	$9/64$
3	$1/64$

28. a) $P(X=2.5)=0$

b) $P(2.5<X<4.5)=\int_{2.5}^{4.5} f(x)\,dx$

$$= \int_{2.5}^{3} \frac{1}{20}x^2\,dx + \int_{3}^{4.5} \frac{33}{2440}(x+x^2)\,dx$$

$$= \frac{1}{60}x^3\Big|_{2.5}^{3} + \frac{33}{2440}\left(\frac{1}{2}x^2+\frac{1}{3}x^3\right)\Big|_{3}^{4.5}$$

$$= \frac{1}{60}(3^3-2.5^3) + \frac{33}{2440}\left[\left(\frac{1}{2}\cdot 4.5^2+\frac{1}{3}\cdot 4.5^3\right)-\left(\frac{1}{2}\cdot 3^2+\frac{1}{3}\cdot 3^3\right)\right]$$

$$\approx 0.55$$

c) $P(X>4.5) = \int_{4.5}^{\infty} f(x)\,dx$

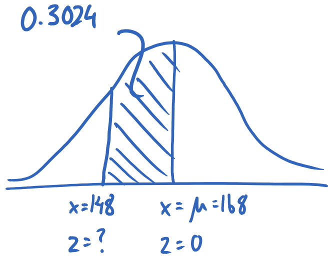
$$= \int_{4.5}^{5} \frac{33}{2440}(x+x^2)\,dx$$

$$= \frac{33}{2440}\left(\frac{1}{2}x^2+\frac{1}{3}x^3\right)\Big|_{4.5}^{5}$$

$$= \frac{33}{2440}\left[\left(\frac{1}{2}\cdot 5^2+\frac{1}{3}\cdot 5^3\right)-\left(\frac{1}{2}\cdot 4.5^2+\frac{1}{3}\cdot 4.5^3\right)\right]$$

$$\approx 0.18$$

29. we want σ such that $P(148 < X < 168) = 0.3024$



reverse look-up:

area = 0.3024 $\Rightarrow$ $z = 0.85$

but $z<0$ so $z = -0.85$

$$z = \frac{x-\mu}{\sigma}$$

$$-0.85 = \frac{148 - 168}{\sigma}$$

$$\sigma = \frac{-20}{-0.85} = 23.5 \text{ lbs}$$

30. we want $P(\bar{x} < 68 \text{ or } \bar{x} > 73)$

$\mu = 71$ $\sigma = 6$ $n = 35$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

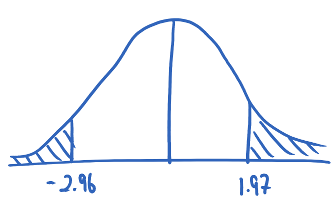
$\bar{x} = 68 \Rightarrow z = \frac{68-71}{6/\sqrt{35}} = -2.96$

$\bar{x} = 73 \Rightarrow z = \frac{73-71}{6/\sqrt{35}} = 1.97$

$P(\bar{x} < 68 \text{ or } \bar{x} > 73) = P(\bar{x} < 68) + P(\bar{x} > 73)$

$= P(z < -2.96) + P(z > 1.97)$

=



$= 1 - 0.4985 - 0.4756 = 0.0259$

31. $n = 50$ $\quad \bar{x} = 30$ $\quad s = 20$

$\sigma \approx s$

a) $C = 0.95 \Rightarrow z_{\alpha/2} = 1.960$

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 30 \pm 1.960\left(\frac{20}{\sqrt{50}}\right) = 30 \pm 5.5$$

so $24.5 < \mu < 35.5$ min

b) $ME = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 5.5$ min

c) $ME = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 1.960\left(\frac{10}{\sqrt{50}}\right) \approx 2.8$ min

32. let x = temperature and y = number of passengers

a) using the calculator: $r = 0.9984$

b) using the calculator: $\hat{y} = 4.413 + 3.953x$

c) $\text{slope} = 3.953 = \frac{3.953}{1} = \frac{\Delta y}{\Delta x}$

As the temperature increases by 1°F, the number of passengers increases by approximately 4.

d) $x = 45 \Rightarrow \hat{y} = 4.413 + 3.953(45) \approx 182$

e) $\Delta y = y - \hat{y}$

$= 173 - 170$

$= 3$

$x = 42 \Rightarrow \hat{y} = 4.413 + 3.953(42) \approx 170$

f) No, because 65 is outside the data set range $30 \leq x \leq 50$