Math 193: Extra Practice for Chapter 29
Solutions

1. For the function $f(x,y) = x^2 \cos 4y$, evaluate the following.

a) $\left.\dfrac{\partial f}{\partial x}\right|_{(3,\pi)} = 2x \cos 4y \Big|_{(3,\pi)} = 2 \cdot 3 \cos 4\pi = 6$

b) $\left.\dfrac{\partial f}{\partial y}\right|_{(3,\pi)} = -4x^2 \sin 4y \Big|_{(3,\pi)} = -4 \cdot 3 \cdot \sin 4\pi = 0$

2. See next page.

3. Find all of the second partial derivatives (all four of them) for the following function.

$f(x,y) = \dfrac{\sin 3y}{1+x^2}$

$$\frac{\partial f}{\partial x} = -\frac{\sin 3y}{(1+x^2)^2} \cdot 2x = -\frac{2x \sin 3y}{(1+x^2)^2}$$ (chain rule)

$$\frac{\partial f}{\partial y} = \frac{3\cos 3y}{1+x^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-2\sin 3y}{(1+x^2)^2} - \frac{(-2)\cdot 2x \sin 3y}{(1+x^2)^3} \cdot 2x$$ (product rule)

$$\boxed{= \frac{-2\sin 3y}{(1+x^2)^2} + \frac{8x^2 \sin 3y}{(1+x^2)^3}} = \frac{2(3x^2-1)\sin 3y}{(1+x^2)^3}$$ if you insist

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{3\cos 3y}{1+x^2}\right) = \frac{-3\cos 3y}{(1+x^2)^2} \cdot 2x = \boxed{\frac{-6x\cos 3y}{(1+x^2)^2}}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{-2x\sin 3y}{(1+x^2)^2}\right) = \boxed{\frac{-6x\cos 3y}{(1+x^2)^2}}$$

look! they're the same!

$$\frac{\partial^2 f}{\partial y^2} = \boxed{\frac{-9\sin 3y}{1+x^2}}$$

2. Find the first partial derivatives of the following function with respect to each of the independent variables. (2 points)

$f(r,t)=4r^2+r\ln(t^3)$ $= 4r^2 + 3r\ln t$

$\boxed{\dfrac{\partial f}{\partial r} = 8r + \ln t^3 \qquad \dfrac{\partial f}{\partial t} = \dfrac{3r}{t}}$

4. Evaluate. (3 points)

$\int_1^2\int_0^{\sqrt{y}}\left(2xy+y^2\right)dx\,dy$

$= \int_1^2 \left(x^2y + xy^2\right)\Big|_0^{\sqrt{y}}\,dy$

$= \int_1^2 \left(y\cdot y + \sqrt{y}\cdot y^2\right)dy$

$= \int_1^2 \left(y^2 + y^{5/2}\right)dy$

$= \left(\frac{y^3}{3} + \frac{2}{7}y^{7/2}\right)\Big|_1^2$

$= \left(\frac{8}{3} + \frac{2}{7}2^{7/2}\right) - \left(\frac{1}{3}+\frac{2}{7}\right)$

$\boxed{= \frac{43}{21} + \frac{16\sqrt{2}}{7} \approx 5.28}$

note: if said $y^2y^{1/2} = y^1$, and $y^{9/2}$, get $23/6$ $\left(-\frac{1}{2}\right)$

if said $y^2y^{1/2} = y^{3/2}$, get $\frac{29}{15} + \frac{8\sqrt{2}}{5}$ ≈ 4.196

5. Integrate the following.

$$\int_0^{\pi/2}\int_0^{\sin y} e^{2x}\cos y\, dx\, dy$$

first, integrate with respect to x

$$\int_0^{\pi/2}\left[\int_0^{\sin y} e^{2x}\cos y\, dx\right]dy$$

$$= \int_0^{\pi/2}\left[\tfrac{1}{2}e^{2x}\cos y\,\Big|_{x=0}^{x=\sin y}\right]dy$$

$$= \int_0^{\pi/2}\left[\tfrac{1}{2}e^{2\sin y}\cos y - \tfrac{1}{2}e^{0}\cos y\right]dy$$

$$= \int_0^{\pi/2}\left[\tfrac{1}{2}e^{2\sin y}\cos y - \tfrac{1}{2}\cos y\right]dy$$

now integrate with respect to y:

$$= \left[\tfrac{1}{4}e^{2\sin y} - \tfrac{1}{2}\sin y\right]\Big|_0^{\pi/2}$$

$$= \left(\tfrac{1}{4}e^{2} - \tfrac{1}{2}\right) - \left(\tfrac{1}{4} - 0\right)$$

$$\boxed{= \tfrac{1}{4}e^{2} - \tfrac{3}{4} \approx 1.097}$$

6. Find the first-octant volume under the plane $z = x + y$ and inside the cylinder $x^2 + y^2 = 9$ using **either** rectangular or cylindrical coordinates.

(Hint: cylindrical is slightly easier once the integral's been set up.)

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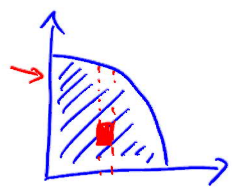
rectangular: (cylindrical's on next page)

region R



$dA = dy\,dx$ or $dx\,dy$

$0 \le x \le 3$

$0 \le y \le \sqrt{9-x^2}$ ← so this is the inside pair of limits and dy inside also

$$V = \iint_R z \, dA$$

$$= \int_0^3 \int_0^{\sqrt{9-x^2}} (x+y)\, dy\, dx$$

can stop here!

$$= \int_0^3 \left[\left(xy + \frac{y^2}{2} \right) \Big|_0^{\sqrt{9-x^2}} \right] dx$$

$$= \int_0^3 \left(x\sqrt{9-x^2} + \frac{9-x^2}{2} \right) dx$$

$$= \left[-\frac{1}{2} \cdot \frac{2}{3} (9-x^2)^{3/2} + \frac{9}{2}x - \frac{x^3}{6} \right] \Big|_0^3$$

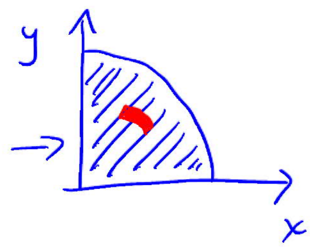
$$= \left(0 + \frac{27}{2} - \frac{27}{6} \right) - \left(-\frac{1}{3} 9^{3/2} \right)$$

$$= 18$$

note: if answer is $\frac{27}{2}$, check that limits on y not $0 \le y \le x$ (−2)

note: if answer is 9, check that lower limit was assumed to be 0 (−1) cylindrical:

region R



$dA = r\,dr\,d\theta$

$0 \le r \le 3$

$0 \le \theta \le \pi/2$

$$V = \iint_R z\, dA$$

$$= \int_0^{\pi/2} \int_0^3 (x+y)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^3 (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

CAN STOP HERE

$$= \int_0^{\pi/2} \int_0^3 r^2 (\cos\theta + \sin\theta)\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[\frac{r^3}{3} (\cos\theta + \sin\theta) \Big|_0^3 \right] d\theta$$

$$= \int_0^{\pi/2} 9 (\cos\theta + \sin\theta)\, d\theta$$

$$= 9 (\sin\theta - \cos\theta) \Big|_0^{\pi/2}$$

$$= 9(1-0) - 9(0-1)$$

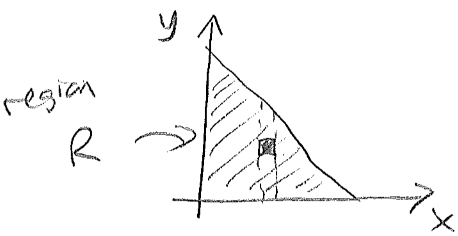
$$= 18$$

if answer is 18, check to see if

$dA = dr\,d\theta$

r is missing

7. Find the first-octant volume below the surface $z = 3x^2$ and bounded by the plane $x + y = 3$. (5 points)



$dA = dx\,dy$ or $dy\,dx$

$0 \le x \le 3$

$0 \le y \le 3 - x$

(2)

see next page for solution with limits switched

$$V = \iint_{\text{region } R} z \, dA$$

$$= \int_0^3 \int_0^{3-x} 3x^2 \, dy \, dx \quad (1)$$

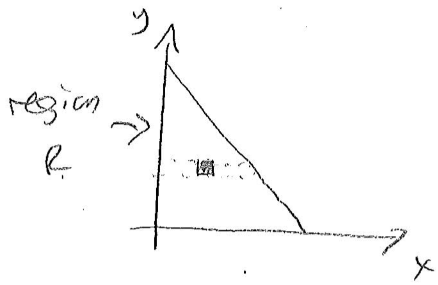
$$= \int_0^3 \left[3x^2 y \Big|_0^{3-x} \right] dx$$

$$= \int_0^3 3x^2(3-x)\, dx \quad (1)$$

$$= \int_0^3 (9x^2 - 3x^3)\, dx$$

$$= \left(3x^3 - \frac{3}{4}x^4\right)\Big|_0^3$$

$$= 3 \cdot 27 - \frac{3 \cdot 81}{4} \quad \boxed{= \frac{81}{4} = 20.25} \quad (1)$$



$dA = dx\,dy$ or $dy\,dx$

$0 \le y \le 3$

$0 \le x \le 3-y$

$$V = \iint_{\substack{\text{region}\\ R}} z \, dA$$

$$= \int_0^3 \int_0^{3-y} 3x^2 \, dx \, dy$$

$$= \int_0^3 \left(x^3 \Big|_0^{3-y} \right) dy$$

$$= \int_0^3 (3-y)^3 \, dy$$

$$= -\frac{(3-y)^4}{4} \Big|_0^3$$

$$= 0 + \frac{3^4}{4} = \boxed{\frac{81}{4} = 20.25}$$