According to the Vancouver Canucks' website (www.canucks.com), the number of goals scored by their top ten scorers in a past year are as follows: (5 points)

36, 24, 23, 20, 18, 14, 12, 12, 12, 11.

a) State the mean, median, of this data set.

mean: 18.2 (1)

median = average of 18 + 14 median: 16 (1)

b) Suppose that in the next game or games, the highest and lowest numbers of goals (36 and 11) each increased by two while all of the other data points stayed the same. What would happen to the median and the range? Be as specific as you can!

=> 38, 24, 23, 20, 18, 14, 13, 12, 12, 12

median still same (1)

mean is now 18.6

(1)

Consider the following sets of data. Without calculating any values, indicate which set will have the higher standard deviation (or will they be the same?). (2 points)

a) Set 1: 1, 3, 5, 7, 9 Set 2: 1, 2, 3, 4, 5

all closer to mean

b) Set 1: 1, 3, 5, 7, 9 Same Set 2: 11, 13, 15, 17, 19

c) Set 1: 1, 4, 5, 6, 9 Set 2: 3, 4, 5, 6, 7

1 + 9 further away

d) Set 1: 22, 23, 24, 25, 26 Set 2: 15, 20, 25, 30, 35

more spread

½ each

The Gizmo Store is having a sale of its Bluetooth-enabled widgets which range in price from $25 to $75. Answer the following questions, being as specific as you can!

a) If every widget is reduced in price by $10, what happens to the mean, median, ~~range~~, and standard deviation of the widget prices?

mean & median decrease by $10

~~range~~ & std dev stay the same

2

b) If, instead, the ~~most~~ least expensive widget is reduced in price by $10, what happens to the mean, median, ~~range~~, and standard deviation of the widget prices?

mean - decreases

median - will stay the same

std dev - increases

2

c) If, instead, every widget is reduced in price by 10%, what happens to the mean, median, ~~range~~, and standard deviation of the widget prices?

everything decreases by 10%

1

An individual is presented with three different glasses of soft drink, labeled A, B, and C. He is asked to taste all three and then list them in order of preference. Suppose that the same soft drink has actually been put into all three glasses.

(a) How many simple events are there in this experiment? What probability would you assign to each event?

(b) What is the probability that A is ranked first?

(c) What is the probability that either B or C is ranked first?

(d) What is the probability that A is ranked first or B is ranked last?

a)

method 1: sample space

ABC BCA CBA
ACB BAC CAB

method 2: multiplication

$\underline{}\ \underline{}\ \underline{}$
3 2 1

so six simple events each with probability $1/6$

b)

method 1:
2 events in sample space (ABC, ACB)
so $2 \cdot 1/6 = 1/3$

method 2:
$\underline{A}\ \underline{}\ \underline{}$
2 1

so $P(A\text{ first}) = 2/6 = 1/3$

c)

method 1: sample space: $P(B\text{ or }C\text{ first}) = \frac{4}{6} = 2/3$ since 4 events have B or C first

method 2: $P(B\text{ or }C\text{ first}) = 1 - P(A\text{ first})$
$= 1 - 1/3 = 2/3$
since $P(A\text{ first})$ was calculated in part (b)

method 3: $P(B\text{ or }C\text{ first}) = P(B\text{ first}) + P(C\text{ first}) - P(B\text{ and }C\text{ first})$
$= 1/3 + 1/3 - 0 = 2/3$

d) $P(A\text{ first or }C\text{ last}) = P(A\text{ first}) + P(B\text{ last}) - P(A\text{ first and }B\text{ last})$
$=$ " $+$ " $- P(ACB)$
$= 1/3 + 1/3 - 1/6 = 1/2$

Students either from the Computing Systems Technology program or from the English department were asked who is the greatest fictional wizard ever, with the following results.

	Gandalf	Dumbledore	
CST	77	63	140
English	33	27	60
	110	90	200

(a) What is the probability that a randomly selected student chose Gandalf?

(b) What is the probability that a randomly selected student studies CST?

(c) What is the probability that a randomly selected ~~English~~ student studies English and chose Dumbledore?

(d) What is the probability that a randomly selected student neither studies English nor chose Dumbledore?

a) $P(G) = \dfrac{n(G)}{n_{TOT}} = \dfrac{110}{200} = 55\%$

b) $P(CST) = \dfrac{n(CST)}{n_{TOT}} = \dfrac{140}{200} = 70\%$

c) $P(E \text{ and } D) = \dfrac{n(E \text{ and } D)}{n_{TOT}} = \dfrac{27}{200} = 13.5\%$

d) $P(\text{not } E \text{ and not } D) = P(CST \text{ and } G) = \dfrac{n(CST \text{ and } G)}{n_{TOT}}$

$= \dfrac{77}{200} = 38.5\%$

A computer system requires a case-sensitive, alpha-numeric password containing six characters.

(a) How many passwords are there that contain no "A"s?

(b) How many passwords are there that contain no "a"s?

(c) How many passwords are there that contain no "A"s ~~or "a"s~~? or no "a"s or both?

case sensitive ↑ lower case different than upper case

alpha-numeric: A-Z, a-z, 0-9

$26+26+10 = 62$ characters

a) no As: $n(\text{no "A"}) = 61 \cdot 61 \cdot 61 \cdot 61 \cdot 61 \cdot 61$

$= 61^6$

if you insist, $= 51{,}520{,}374{,}361$

b) Same as (a), so

$n(\text{no "a"}) = 61^6$

c) $n(\text{no "A" or no "a" or both}) = n(\text{no "A"}) + n(\text{no "a"}) - n(\text{both})$

but $n(\text{no "A" and no "a"}) = 60^6$

so $n(\text{no "A" or no "a" or both}) = 61^6 + 61^6 - 60^6$

$= 56\ 384\ 748\ 722$

Consider the random variable x with the following probability distribution:

x	$p(x)$
5	0.25
10	0.65
15	0.1

because the sum of this column must be one

(a) Complete the table above by filling in the missing entry.

(b) Calculate the mean and standard deviation of x.

b)

$$\mu = E(x) = \sum x\, p(x)$$

$$= 5(0.25) + 10(0.65) + 15(0.1)$$

$$= 9.25$$

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

$$= 5^2(0.25) + 10^2(0.65) + 15^2(0.1) - 9.25^2$$

$$= 8.1875$$

$$\sigma = \sqrt{\sigma^2} \approx 2.86138$$

$$\approx 2.86$$

You can insure a \$50,000 diamond for its total value by paying a premium of \$1250. If the insurance company's expected gain is \$1000 per insurance policy, what is the probability of theft for this diamond?

let x = company's income

let y = probability of theft

x	$P(x)$
1250	$1-y$ (no theft)
$1250-50000$	y (theft)

$$E(x) = \sum x\,p(x)$$

$$1000 = 1250(1-y) + (1250-50000)y$$

$$= 1250(1-y) + 1250y - 50000y$$

$$1000 = 1250 - 50000y$$

$$50000y = 250$$

$$y = \frac{250}{50000} = 0.005$$

The probability of theft is 0.5%

For each of the following experiments, state whether it is a binomial experiment by writing Y (yes) or N (no).

(a) Counting the number of buses that stop at your bus stop before the one with the route you want shows up. N

(b) Standing by the coffee kiosk and, for the next ten people, noting whether that person brought their own mug. Y

(c) Surveying 100 random students at lunch to ask whether they got to school by driving, by taking the bus, or by biking/walking. N

(d) Flipping a fair coin and recording the results, stopping once youve got three heads in a row. N

(e) Flipping an unfair coin and recording the results for 20 trials. Y

The average number of dandelions in Pat's front lawn is six. (3 points)

a) Let x be the number of dandelions found in her lawn today. What is the name of the probability distribution that would best describe x?

Poisson (number of events in given space/time)

b) Using the distribution you've chosen, calculate the probability that there is more than one dandelion in Pat's front lawn today.

$P(x=k) = \frac{\mu^k e^{-\mu}}{k!}$ so

$$P(x>1) = 1 - P(x=0) - P(x=1)$$

$$= 1 - \frac{6^0 e^{-6}}{0!} - \frac{6^1 e^{-6}}{1!}$$

$$= 0.982649$$

$$= \boxed{98\%}$$

An airline finds that, on average, 5% of the persons making reservations on a certain flight will not show up. If a Dash 8 making the flight from Victoria to Seattle only seats 50 passengers, calculate the following probabilities.

(a) Calculate the probability that the plane will be completely full.

(b) Calculate the probability that there will be at least one empty seat.

(c) Calculate the probability that there will be exactly two empty seats.

this is binomial with $n = 50$

$p = 0.05$ (probability of empty seat)

let $x =$ number of empty seats

$$P(x = k) = {}_nC_k\, p^k q^{n-k}$$

a) $P(x=0) = {}_{50}C_0\,(0.05)^0\,(0.95)^{50-0}$

$= 0.076945$

$= 7.7\%$

b) $P(x \geq 1) = 1 - P(0) = 1 - 0.076945$

$= 0.923055 = 92.3\%$

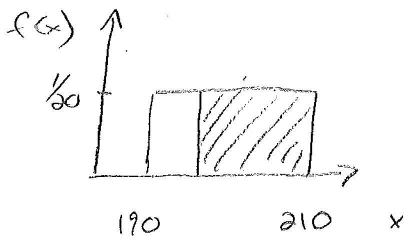
c) $P(x=2) = {}_{50}C_2\,(0.05)^2\,(0.95)^{50-2}$

$= 0.261101$

$= 26.1\%$

A soft-drink machine is regulated so that it discharges an amount of liquid which is a uniform random variable with values between 190 and 210 mL. (3 points)

a) Calculate the fraction of drinks dispensed that will have a volume greater than 195 mL.



$P(x > 195) = \boxed{75\%}$ (area of shaded region = 3/4 of total area)

b) Calculate the mean and standard deviation of the volume of liquid this machine dispenses.

$\boxed{\mu = 200 \text{ mL}}$ (centre of the rectangle)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{190}^{210} x^2 \cdot \frac{1}{20}\,dx - 200^2$$

$$= \frac{1}{60} x^3 \Big|_{190}^{210} - 200^2$$

$$= \frac{1}{60}\left(210^3 - 190^3\right) - 200^2$$

$$= 33.\bar{3}$$

so $\sigma \approx 5.7735$ mL

$\boxed{\sigma \approx 6 \text{ mL}}$

Suppose that some phenomenon has the following probability distribution.

$$f(x) = \begin{cases} \frac{k}{1+x^2} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the following, giving your answer to three decimal places.

(a) Calculate k so that $f(x)$ is indeed a probability distribution function.

(b) Calculate the probability of x being between $\frac{1}{\sqrt{3}}$ and 1.

(c) Calculate the mean value of x.

a) $1 = \int_{-\infty}^{\infty} f(x)\,dx$

$= \int_0^1 \frac{k\,dx}{1+x^2}$

$= k \tan^{-1} x \Big|_0^1$

$1 = k \cdot \pi/4$

so $k = 4/\pi$

≈ 1.273

b) $P\left(\frac{1}{\sqrt{3}} < x < 1\right) = \int_{1/\sqrt{3}}^{1} \frac{4}{\pi(1+x^2)}\,dx$

$= \frac{4}{\pi} \tan^{-1} x \Big|_{1/\sqrt{3}}^{1}$

$= \frac{4}{\pi}\left(\pi/4 - \pi/6\right)$

$= 1/3$ or 0.333

c) $\mu = E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

$= \int_0^1 x \,\frac{4}{\pi}\, \frac{dx}{1+x^2}$

let $u = 1+x^2$

$du = 2x\,dx$

$= \frac{4}{\pi} \int_1^2 \frac{1}{2} \frac{du}{u}$

$= \frac{2 \ln|u|}{\pi} \Big|_1^2$

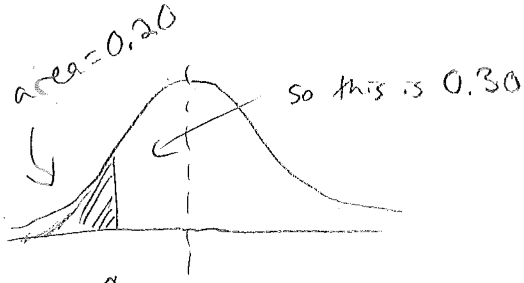
$\mu = \frac{2}{\pi}\left(\ln 2 - \cancel{\ln 1}^{\,0}\right)$

$= \frac{2 \ln 2}{\pi}$ or $\frac{\ln 4}{\pi}$

≈ 0.441

The mayor of Victoria was informed that household water usage is a normally distributed random variable with mean of 25 gallons/day and a standard deviation of 6 gallons/day. (4 points)

a) If the mayor wants to give a tax rebate to the lowest 20% of water users, what should the gallons/day cutoff be?



$z = -0.84$

from normal table

$$z = \frac{x-\mu}{\sigma}$$

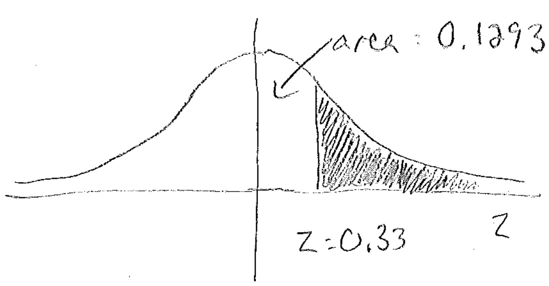
$$x = \mu + z\sigma$$
$$= 25 + (-0.84)(6)$$
$$= 19.96$$
$$\approx 20$$

2

The cutoff should be 20 gallons/day.

b) Calculate the probability that a randomly-chosen household will use more than 27 gallons per day.

$$z = \frac{x-\mu}{\sigma} = \frac{27-25}{6} = 0.33$$



$$P(z > 0.33) = 0.5 - 0.1293$$
$$= 0.3707$$

There is a 37% probability that a randomly-chosen household will exceed 27 gallons per day.

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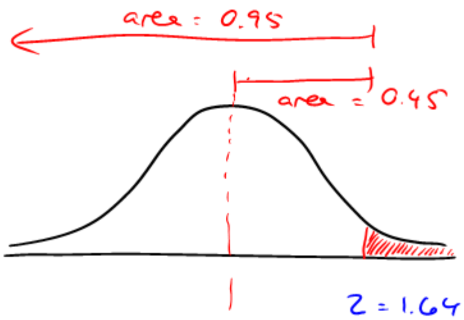

→ Corresponds to $z = 1.645$

$z = 1.645$

(1.64 and 1.65 also acceptable)

$$z = \frac{x - \mu}{\sigma}$$

$$\mu = x - z\sigma$$

$$= 75000 - (1.645)(5000)$$

$$= 66775$$

(66800 if used $z = 1.64$, 66750 if use $z = 1.65$)

The publisher should specify 67,000 words.