Math 193 – Test 1: Version A

February 2, 2018 **Name:** Solution Set
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Total: 25 points

Integrate the following.

1. (3 points) $\displaystyle\int \frac{3y}{\sqrt{4-y^2}}\,dy$

let $u = 4 - y^2$
$du = -2y\,dy$
$\frac{du}{-2} = y\,dy$

$$= \int \frac{3}{-2} u^{-1/2}\,du$$

$$= -\frac{3}{2} \cdot 2u^{1/2} + C$$

$$\boxed{= -3\sqrt{4-y^2} + C}$$

coeff/sign problem $-\frac{1}{2}$ each
didn't substitute back -1
used arcsin form -3
no $+C$ $-\frac{1}{2}$

2. (3 points) $\displaystyle\int \frac{e^{\tan\theta}}{\cos^2\theta}\,d\theta$

let $u = \tan\theta$
$du = \sec^2\theta\,d\theta$

$$= \int \sec^2\theta\, e^{\tan\theta}\,d\theta$$

$$= \int e^u\,du$$

$$= e^u + C$$

$$\boxed{= e^{\tan\theta} + C}$$

maximum of -1 for entire test on $+C$

3. (4 points) $\displaystyle\int \frac{dx}{x^2 - 6x + 25}$

$$= \int \frac{dx}{(x-3)^2 + 4^2}$$

$$\boxed{= \frac{1}{4} \tan^{-1}\left(\frac{x-3}{4}\right) + C}$$

completing the square:

$$x^2 - 6x + 25 = (x^2 - 6x + 9) + 25 - 9$$
$$= (x-3)^2 + 16$$
$$= (x-3)^2 + 4^2$$

omitted the 4 entirely

-1

4. (4 points) $\displaystyle\int \frac{24}{x^2 - 6x}\, dx$

partial fractions: $\dfrac{24}{x^2 - 6x} = \dfrac{24}{x(x-6)} = \dfrac{A}{x} + \dfrac{B}{x-6}$

$$24 = A(x-6) + Bx$$

let $x = 0$:

$$24 = -6A$$
$$A = -4$$

let $x = 6$:

$$24 = 6B$$
$$B = 4$$

incorrect factoring

-1

So $\displaystyle\int \frac{24}{x^2 - 6x}\, dx = \int \left(-\frac{4}{x} + \frac{4}{x-6}\right) dx$

$$\boxed{= -4 \ln|x| + 4 \ln|x-6| + C}$$

5. (4 points) $\int e^x \cos 2x \, dx$

D		I
$\cos 2x$	$+$	e^x
$-2\sin 2x$	$-$	e^x
$-4\cos 2x$	$+$	e^x

$$\int e^x \cos 2x \, dx = e^x \cos 2x - (-2\sin 2x)e^x + \int (-4\cos 2x)e^x \, dx$$

$$= e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x \, dx$$

move this to LHS

$$5\int e^x \cos 2x \, dx = e^x \cos 2x + 2e^x \sin 2x$$

$$\boxed{\int e^x \cos 2x \, dx = \frac{1}{5} e^x \cos 2x + \frac{2}{5} e^x \sin 2x + C}$$

(1/2) correct method
(1) first integral
(1) second integral
(1) moving $\int e^x \cos 2x$ to LHS
(1/2) correct coefficients

check: $\frac{d}{dx}\left(\frac{1}{5}e^x \cos 2x + \frac{2}{5}e^x \sin 2x + C\right)$

product rule

$$= \frac{1}{5}e^x \cos 2x - \frac{2}{5}e^x \sin 2x + \frac{2}{5}e^x \sin 2x + \frac{4}{5}e^x \cos 2x$$

$$= \frac{1}{5}e^x \cos 2x + \frac{4}{5}e^x \cos 2x$$

$$= e^x \cos 2x \quad \checkmark$$

6. (4 points) Find the two first partial derivatives for the following function.

$$f(x,y) = \frac{e^{-x}}{y} + 5\ln x$$

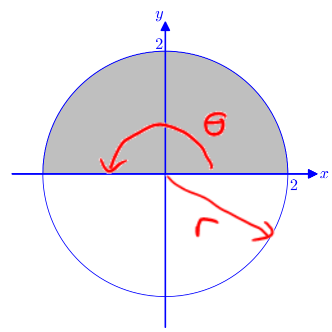
$$\frac{\partial f}{\partial x} = \frac{-e^{-x}}{y} + \frac{5}{x}$$

$$\frac{\partial f}{\partial y} = \frac{-e^{-x}}{y^2}$$

$-\frac{1}{2}$ incorrect notation

7. (3 points) Using polar coordinates, set up BUT DO NOT EVALUATE the double integral for the volume of the solid with vertical sides, a top surface of $z = 3 - x$ and a base in the xy-plane consisting of a semicircle of radius 2.

Hint: Use $V = \iint z \, dA$.



$0 \leq r \leq 2$

$0 \leq \theta \leq \pi$

$$V = \int_0^{\pi} \int_0^{2} z \, r\,dr\,d\theta$$

$$= \int_0^{\pi} \int_0^{2} (3-x)\, r\,dr\,d\theta$$

$$\boxed{V = \int_0^{\pi} \int_0^{2} (3 - r\cos\theta)\, r\,dr\,d\theta}$$

(-3) rectangular coords

(-1) leaving z in terms of x/y

(-1) forgetting r in rdrdθ