Math 193 – Test 1: Version B

February 2, 2018 Name: Solution Set
Instructor: Patricia Wrean

Total: 25 points

Integrate the following.

1. (3 points) $\int \frac{5z}{(1-z^2)^2}\,dz$

$= \int \frac{5}{-2} u^{-2}\,du$

$= -\frac{5}{2}\frac{u^{-1}}{-1} + C$

$\boxed{= \frac{5}{2(1-z^2)} + C}$

let $u = 1-z^2$

$du = -2z\,dz$

$\frac{du}{-2} = z\,dz$

coeff/sign problem $-\frac{1}{2}$ each

didn't substitute back -1

used $\ln|u^2|$ instead -1

no $+C$ $-\frac{1}{2}$

2. (3 points) $\int \frac{e^{\cos\theta}}{\csc\theta}\,d\theta$

$= \int \sin\theta\, e^{\cos\theta}\,d\theta$

$= \int -e^u\,du$

$= -e^u + C$

$= -e^{\cos\theta} + C$

let $u = \cos\theta$

$du = -\sin\theta\,d\theta$

$-du = \sin\theta\,d\theta$

maximum of -1 for entire test on $+C$

3. (4 points) $\displaystyle\int \frac{24}{x^2-8x}\,dx$

partial fractions: $\dfrac{24}{x^2-8x} = \dfrac{24}{x(x-8)} = \dfrac{A}{x} + \dfrac{B}{x-8}$

$24 = A(x-8) + Bx$

let $x=0$:
$24 = -8A$
$A = -3$

let $x=8$:
$24 = 8B$
$B = 3$

so $\displaystyle\int \frac{24}{x^2-8x}\,dx = \int \left(\frac{-3}{x} + \frac{3}{x-8}\right)dx = \boxed{-3\ln|x| + 3\ln|x-8| + C}$

incorrect factoring
-1

4. (4 points) $\displaystyle\int \frac{dx}{x^2-8x+25}$

completing the square:
$x^2-8x+25 = (x^2-8x+16)+25-16$
$= (x-4)^2+9$
$= (x-4)^2+3^2$

$\displaystyle\int \frac{dx}{x^2-8x+25}$

$\displaystyle = \int \frac{dx}{(x-4)^2+3^2}$

$\displaystyle = \frac{1}{3}\arctan\left(\frac{x-4}{3}\right)+C$

$\displaystyle = \frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right)+C$

omitted the 3 entirely
-1

5. (4 points) $\int e^{2x} \sin x \, dx$

	O	I
	$\sin x$	e^{2x}
$\oplus$	$\cos x$	$\frac{1}{2}e^{2x}$
$\ominus$	$-\sin x$	$\frac{1}{4}e^{2x}$
$\oplus$		

$$\int e^{2x}\sin x\,dx = \tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x + \int \tfrac{1}{4}e^{2x}(-\sin x)\,dx$$

$$= \tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x - \tfrac{1}{4}\int e^{2x}\sin x\,dx$$

move to LHS

$$\tfrac{5}{4}\int e^{2x}\sin x\,dx = \tfrac{1}{2}e^{2x}\sin x - \tfrac{1}{4}e^{2x}\cos x$$

$$\boxed{\int e^{2x}\sin x\,dx = \tfrac{2}{5}e^{2x}\sin x - \tfrac{1}{5}e^{2x}\cos x + C}$$

- (1/2) correct method
- (1) first integral
- (1) second integral
- (1) moving $\int e^{2x}\sin x$ to LHS
- (1/2) correct coefficients

check: $\frac{d}{dx}\left(\frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C\right)$

product rule

$$= \tfrac{4}{5}e^{2x}\sin x + \tfrac{2}{5}e^{2x}\cos x - \tfrac{2}{5}e^{2x}\cos x + \tfrac{1}{5}e^{2x}\sin x$$

$$= \tfrac{4}{5}e^{2x}\sin x + \tfrac{1}{5}e^{2x}\sin x$$

$$= e^{2x}\sin x \quad \checkmark$$

6. (4 points) Find the two first partial derivatives for the following function.

$$f(x,y) = \frac{\ln x}{y} - 4\sqrt{y}$$

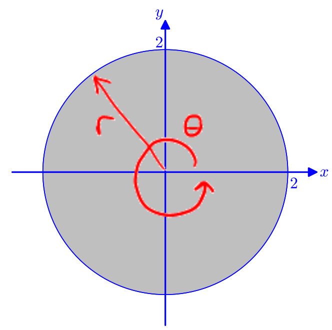
$$\frac{\partial f}{\partial x} = \frac{1}{xy}$$

$$\frac{\partial f}{\partial y} = -\frac{\ln x}{y^2} - \frac{2}{\sqrt{y}}$$

($-\frac{1}{2}$) incorrect notation

7. (3 points) Using polar coordinates, set up BUT DO NOT EVALUATE the double integral for the volume of the solid with vertical sides, a top surface of $z = 6 - xy$ and a base in the xy-plane consisting of a circle of radius 2.

Hint: Use $V = \iint z\, dA$.



$$0 \le r \le 2$$

$$0 \le \theta \le 2\pi$$

$$V = \iint z\, dA$$

$$= \int_0^{2\pi}\int_0^2 z\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 (6 - xy)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 (6 - r\sin\theta\, r\cos\theta)\, r\,dr\,d\theta$$

$$\boxed{= \int_0^{2\pi}\int_0^2 (6 - r^2\sin\theta\cos\theta)\, r\,dr\,d\theta}$$

(-3) rectangular coords

(-1) leaving z in terms of x/y

(-1) forgetting r in rdrdθ