Math 193 – Test 3: Version A

March 12, 2018 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 25 points

1. (3 points) There are 30 players on the Vancouver Whitecaps team. The top two player salaries are \$1,400,000, and \$725,000.

   What would happen to the mean, median and standard deviation of the player salaries if the highest paid player's salary was decreased to \$1,000,000? Circle the correct answer.

mean:	increase	decrease	no change
median:	increase	decrease	no change
SD:	increase	decrease	no change

← the middle salary is unchanged

2. (5 points) An experiment consists of flipping a coin and then rolling a fair six-sided die.

   (a) How many possible outcomes does this experiment have?

method #1:

$\underline{2}\ \underline{6} = \boxed{12 \text{ outcomes}}$

method #2:

$\boxed{12}$

sample space:

H1 H2 H3 H4 H5 H6
T1 T2 T3 T4 T5 T6

2

   (b) What is the probability that the coin toss is *TAILS* and the die roll is a *5*?

$\underline{1}\ \underline{1} = 1$ outcome

$P(E) = \frac{n(E)}{n(S)}$

$\boxed{P(T5) = \frac{1}{12} = 8.\bar{3}\%}$

$P(E) = \frac{n(E)}{n(S)}$

$\boxed{P(T5) = \frac{1}{12}}$

1

   (c) What is the probability that the coin toss is *TAILS* or the die roll is a *5*?

$P(T) = \frac{1}{2}$

$P(5) = 1/6$

$P(T5) = 1/12$

$P(T \text{ or } 5) = P(T) + P(5) - P(T5)$

$= \frac{1}{2} + 1/6 - 1/12$

$\boxed{= \frac{7}{12} = 58.3\%}$

$P(T \text{ or } 5) = \frac{7}{12}$

2

3. (3 points) A recent study revealed that 80% of all fish in sold in BC is mislabelled. Suppose you are planning to buy fish 6 times next month. What is the probability that at least one of those six purchases is correctly labelled?

let X = number of fish correctly labelled

1 binomial with $n=6$ and $p=0.2$ (so $q=0.8$)

$P(x) = {}_nC_x\, p^x q^{n-x}$

1 $P(0) = {}_6C_0 (0.2)^0 (0.8)^6 \approx 0.262144$

3

1 $P(x>0) = 1 - P(0) \approx 0.737856$

$\boxed{\approx 74\%}$

4. (3 points) On a particular river, overflow floods occur once every 100 years on average. Calculate the probability that in the next 100 years there will be

(a) exactly one flood

let X = number of floods in 100 years

Poisson with $\mu = 1$ 1

$P(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$

$P(1) = \dfrac{1^1 e^{-1}}{1!} = e^{-1} \approx 0.367879$ 1

$\boxed{\approx 37\%}$

(b) no floods

$P(0) = \dfrac{1^0 e^{-1}}{0!} = e^{-1} = \boxed{\text{same as above}}$ 1

5. (5 points) In a certain city, the time it takes in hours to repair a square of sidewalk is a continuous random variable with probability density function

$$f(x) = \begin{cases} 30(x^4 - x^5) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the probability that it takes exactly half an hour to repair a sidewalk square.

$P(x = 0.5) = 0$

the probability of any single continuous random variable is exactly zero

(b) Find the probability that it takes at least half an hour to repair a sidewalk square.

$P(a < x < b) = \int_a^b f(x)\,dx$

$P(0.5 < x < 1) = \int_{0.5}^1 30(x^4 - x^5)\,dx$

$= \left(\frac{30}{5}x^5 - \frac{30}{6}x^6\right)\Big|_{0.5}^1$

$\boxed{= \frac{57}{64} \approx 0.890625 \approx 89\%}$

(c) On average, how long does it take to repair a sidewalk square?

$\mu = E(x) = \int_{-\infty}^{\infty} x\,f(x)\,dx$

$= \int_0^1 x \cdot 30(x^4 - x^5)\,dx$

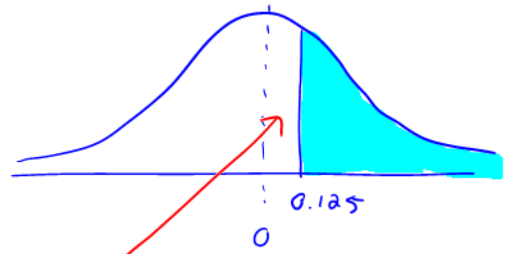
$= \left(\frac{30x^6}{6} - \frac{30x^7}{7}\right)\Big|_0^1$

$= \frac{30}{6} - \frac{30}{7} = \boxed{\frac{5}{7} \text{ hours or } 0.714 \text{ hours}}$

6. (6 points) Suppose that vehicle speeds on the Malahat can be represented with a normal distribution and that mean is 98 km/h while the standard deviation is 16 km/h.

(a) What percentage of vehicle speeds are over 100 km/h?

$z = \frac{x-\mu}{\sigma} = \frac{100-98}{16} = 0.125$



$P(z > 0.125) = 0.5 - 0.04975$
$= 0.45025 = \boxed{45\%}$

$\left[\begin{array}{l} \text{for } z = 0.12,\ P = 0.4522 \\ \quad z = 0.13,\ P = 0.4483 \end{array}\right]$

if you used $z = 0.125$, area $= \frac{1}{2}(0.0478 + 0.0517) = 0.04975$

but $z = 0.12$, area $= 0.0478$
$z = 0.13$, area $= 0.0517$ also acceptable

(b) What percentage of vehicle speeds are under 100 km/h?

$P(z < 0.25) = 1 - P(z > 0.25)$
$= 0.5987$ or $\boxed{59.9\%}$

(c) What speed separates the fastest 10% of all speeds from the slowest 90%?

$z = \frac{x-\mu}{\sigma}$

$x = \mu + z\sigma$
$= 96 + (1.28)(16)$
$= 116.48$
$= \boxed{116 \text{ km/h}}$