(white)

Math 193 – Test #1: Form A

Jan 27, 2017 **Name:** Solution Set
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Total: 25 points

Evaluate the following integrals.

1. $\int_0^1 \frac{6e^x\,dx}{\sqrt{3e^x+2}}$ (~~3~~ 4 points)

let $u = 3e^x + 2$
$du = 3e^x dx$ 1

$= \int_{x=0}^{x=1} \frac{\frac{6}{3}du}{\sqrt{u}}$ 1

$= \int_{x=0}^{x=1} 2u^{-1/2}\,du$

$= 2\left.\frac{u^{1/2}}{1/2}\right|_{x=0}^{x=1}$ 1

$= 4\sqrt{3e^x+2}\Big|_0^1 = \boxed{4\sqrt{3e+2} - 4\sqrt{5}}$ 1

(≈ 3.8024 if you insist)

(−1) problem with coeff $6 \to 2$

(−1) no idea where numerical value comes from

(−½) $x=0$, $x=1$ missing

(−1) major limits problem

2. $\int \frac{24dp}{4p^2+1}$ (3 points)

method #1:

let $u = 2p$
$du = 2dp$

$\int \frac{24\,dp}{4p^2+1} = \int \frac{12\,du}{u^2+1}$

$= 12\tan^{-1}u + C$

$= 12\tan^{-1}2p + C$

method #2:

$\int \frac{24\,dp}{4p^2+1} = \int \frac{24\,dp}{4(p^2+1/4)}$

$= \frac{6}{1/2}\tan^{-1}\frac{p}{1/2} + C$

$= 12\tan^{-1}2p + C$

(−1) major problem with coeff

(−½) $4p$, not $2p$

(−2) only got arctan part

(−3) basic substitution

3. $\int \left(5+e^{-3y}\right)^2 \, dy$ (3 points)

$$= \int \left(25 + 10e^{-3y} + e^{-6y}\right) dy$$

$$\boxed{= 25y - \frac{10}{3}e^{-3y} - \frac{1}{6}e^{-6y} + C}$$

$-\frac{1}{2}$ wrong variable

-3 basic substitution

$-\frac{1}{2}$ negative sign error

-1 chain rule major error ($-\frac{1}{2}$ minor)

$-1\frac{1}{2}$ forgot to integate

4. $\displaystyle\int \frac{\sin 2x}{1-\cos^2 x} \, dx$ (4 points)

$$= \int \frac{2\sin x \cos x}{1-\cos^2 x} \, dx$$

let $u = 1-\cos^2 x$

$du = +2\cos x \sin x$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

$$\boxed{= \ln \left|1-\cos^2 x\right| + C} = \boxed{2\ln |\sin x| + C}$$

5. $\int \frac{e^{3\cot\theta}}{\sin^2\theta}\,d\theta$ (~~4~~ 3 points)

$$= \int \csc^2\theta \, e^{3\cot\theta}\,d\theta$$

let $u = 3\cot\theta$
$du = -3\csc^2\theta\,d\theta$

$$= \int -\frac{1}{3} e^{u}\,du$$

$$= -\tfrac{1}{3} e^{u} + C$$

$$\boxed{= -\tfrac{1}{3} e^{3\cot\theta} + C}$$

(1) correct trig identity
(1) correct substitution to $\int e^u du$
(1) answer

($-2/3$) if only let $u = 3\cot\theta$ with nothing else

($-1/3$) coeff error
($-1/3$) wrong variable

6. $\int \frac{2x+12}{x^2-9}\,dx$ (4 points)

partial fractions:

$$\frac{2x+12}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$2x+12 = A(x+3) + B(x-3)$$

let $x = 3$

$$18 = 6A$$
$$A = 3$$

let $x = -3$.

$$6 = -6B$$
$$B = -1$$

$$\int \frac{2x+12}{x^2-9}\,dx = \int \left[\frac{3}{x-3} - \frac{1}{x+3}\right] dx$$

$$\boxed{= 3\ln|x-3| - \ln|x+3| + C}$$

7. Use integration by parts to evaluate the following integral. You may use the table method if you wish. (4 points)

$\int \sqrt{x} \ln x \, dx$

(1) for u & v

(1½) for first term

(1½) for second integral

method #1:

let $u = \ln x$ $\quad dv = \frac{2}{3}x^{3/2}$

$du = \frac{1}{x}\,dx$ $\quad v = \sqrt{x} = x^{1/2}$

$\int u\,dv = uv - \int v\,du$

$\int \sqrt{x}\ln x\,dx = \ln x\left(\frac{2}{3}x^{3/2}\right) - \int \frac{2}{3}x^{3/2}\frac{1}{x}\,dx$

$= \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2}\,dx$

$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + C$

$\boxed{= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C}$

method #2

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$\ln x$	$x^{1/2}$
$1/x$	$\frac{2}{3}x^{3/2}$

(+) (−)

$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{3/2}\frac{1}{x}\,dx$

$= \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2}\,dx$

etc

(−1½) not being able to integrate $\frac{2}{3}\frac{x^{3/2}}{x}\,dx$

(−3) entries in table but nothing else