Math 193 – Test 3: Version A

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Instructor: Patricia Wrean

Name: Solution Set

Total: 25 points

1. (4 points) Solve $y'' + 4y' + 4y = 0$ if $y(0) = 5$ and $y'(0) = 0$.

auxiliary equation:

$$m^2 + 4m + 4 = 0$$
$$(m+2)^2 = 0$$
$$m = -2 \quad \text{(one repeated root)}$$ 1

$$y = (c_1 + c_2 x)e^{-2x}$$ 1

at $x = 0$, $y = 5$:

$$5 = (c_1 + c_2 \cdot 0)e^0$$
$$5 = c_1$$

so $y = (5 + c_2 x)e^{-2x}$

$$y' = c_2 e^{-2x} - 2(5 + c_2 x)e^{-2x}$$
$$0 = c_2 - 2(5 + 0)$$
$$0 = c_2 - 10$$
$$c_2 = 10$$ 2

$$\boxed{y = (5 + 10x)e^{-2x}}$$

2. (5 points) Solve $y'' - 2y' + 10y = 5x$.

complementary solution: $y'' - 2y' + 10y = 0$

$m^2 - 2m + 10 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$

$= a \pm bi$

$y_c = e^x (c_1 \cos 3x + c_2 \sin 3x)$

particular solution: RHS $= 5x$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$y'' - 2y' + 10y = 5x$

$0 - 2A + 10(Ax+B) = 5x$

$10Ax + (-2A + 10B) = 5x$

$10A = 5$ and $-2A + 10B = 0$

$A = 1/2$ $\quad -1 + 10B = 0$

$B = 1/10$

$y_p = 1/2\, x + 1/10$

full:

$$\boxed{y = y_c + y_p = e^x (c_1 \cos 3x + c_2 \sin 3x) + 1/2\, x + 1/10}$$

3. (5 points) Solve $y'' + 5y' + 4y = 6e^{-4x}$.

complementary : $y'' + 5y' + 4y = 0$

$m^2 + 5m + 4 = 0$

$(m+1)(m+4) = 0$

$m = -1, -4$

$y_c = C_1 e^{-x} + C_2 e^{-4x}$

bad case!

particular. RHS $= 6e^{-4x}$

$y_p = Axe^{-4x}$

$y_p' = Ae^{-4x} - 4Axe^{-4x}$

$y_p'' = -4Ae^{-4x} - 4Ae^{-4x} + 16Axe^{-4x}$

$= -8Ae^{-4x} + 16Axe^{-4x}$

so $y'' + 5y' + 4y = 6e^{-4x}$

$(-8Ae^{-4x} + 16Axe^{-4x}) + 5(Ae^{-4x} - 4Axe^{-4x}) + 4Axe^{-4x} = 6e^{-4x}$

$-8Ae^{-4x} + \cancel{16Axe^{-4x}} + 5Ae^{-4x} - \cancel{20Axe^{-4x}} + \cancel{4Axe^{-4x}} = 6e^{-4x}$

$-3Ae^{-4x} = 6e^{-4x}$

$-3A = 6$

$A = -2$

$y_p = -2xe^{-4x}$

full:

$$\boxed{y = y_c + y_p = C_1 e^{-x} + C_2 e^{-4x} - 2xe^{-4x}}$$

4. (3 points) State the form of the particular solution y_p for the following. Leave your answer with undetermined coefficients. (This means "Write down your initial guess for y_p but dont bother to solve for the constants.") Please note that the complementary solution for the homogeneous equation is $y_c = C_1 + C_2e^{6x}$.

   (a) $y'' - 6y' = -5e^{3x}$ $y_p = Ae^{3x}$

   (b) $y'' - 6y' = 5$ bad case! $y_p = Ax$

   (c) $y'' - 6y' = -3e^x \cos 2x$ $y_p = Ae^x\cos 2x + Be^x \sin 2x$

5. (3 points) Write down the DE and the initial conditions for the following system.

   A 7 kg mass is attached to a spring with spring constant 2.5 N/m. There is a damping force equal to 2 times the velocity, as well as an external force $F(t) = 4\cos 2t$. The mass is initially 0.15 m above the equilibrium position with an downwards velocity of 0.10 m/s.

DO NOT SOLVE THE DE!

$m = 7$ $b = 2$
$k = 2.5$
$F_{ext} = 4\cos 2t$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext}$$

$$\boxed{7\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2.5x = 4\cos 2t}$$

at $t = 0$, $x = 0.15$
$t = 0$, $\frac{dx}{dt} = -0.10$

6. (5 points) Consider the following population of starting salaries in a graduating class of 40 students:

salary	frequency
\$45,000	3
\$50,000	5
\$55,000	10
\$60,000	15
\$65,000	5
\$70,000	1
\$175,000	1

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(a) Find the mean salary.

$$\mu = \frac{\Sigma x}{n} = \frac{45 \cdot 3 + 50 \cdot 5 + 55 \cdot 10 + 60 \cdot 15 + 65 \cdot 5 + 70 \cdot 1 + 175 \cdot 1}{40}$$

$= 60.125$ in thousands of dollars

$\boxed{= \$60125}$ (okay to round to \$60100)

(b) Find the median salary.

$\frac{n+1}{2} = \frac{40+1}{2} = 20.5$ so average of $\underbrace{20^{th} \text{ and } 21^{st}}_{\text{they are both } \$60{,}000}$ point

$\boxed{\text{median} = \$60000}$

(c) After 2 years, each person's salary is increased by \$10,000. How does the new standard deviation compare to the initial standard deviation?

$\boxed{\text{std dev is the same}}$

(if all data points shift by same amount, spread of data is unchanged)