Math 193 – Test 3: Version A

March 17, 2017 | Name: Solution Set

Instructor: Patricia Wrean

Total: 25 points

1. (6 points) On the most recent sailing of the BC Ferry "Spirit of BC", 125 of the vehicles had BC licence plates, 40 were campers, and 5 were campers with BC licence plates. There were 200 vehicles in total on that ferry.

(a) Complete the contingency table below using the above information.

← (−½) each mistake

	camper	not a camper	
BC plates	5	120	125
elsewhere	35	40	75
	40	160	200

2

(b) Calculate the probability that a random vehicle is not a camper.

$$P(\bar{C}) = \frac{n(\bar{C})}{n_{TOT}} = \frac{160}{200} = 80\%$$

1

(c) Calculate the probability that a random vehicle is a camper that does not have BC plates.

$$P(C \,\&\, \overline{BC}) = \frac{n(C \,\&\, \overline{BC})}{n_{TOT}} = \frac{35}{200} = 17.5\%$$

1

(d) Calculate the probability that a random vehicle is a camper or has BC plates or both.

$$P(C \text{ or } BC) = \frac{n(C \text{ or } BC)}{n_{TOT}} = \frac{5+120+35}{200} = \frac{160}{200} = 80\%$$

2

or $P(C \text{ or } BC) = P(C) + P(BC) - P(\text{both})$

$$= \frac{40}{160} + \frac{125}{160} - \frac{5}{160} = \frac{160}{200} = 80\%$$

or $P(C \text{ or } BC) = 1 - P(\bar{C} \,\&\, \overline{BC}) = 1 - \frac{40}{160} = 80\%$

2. (4 points) A skilled marksman hits a target 91% of the time. He does not improve with practice. What is the probability that he hits the target at most 17 times on 20 attempts?

binomial with $n = 20$

$p = 0.91$

1

$$P(x=k) = {}_nC_k \, p^k q^{n-k}$$

$$P(x=18) = {}_{20}C_{18} (0.91)^{18} (0.09)^2 = 0.28182$$

$$P(x=19) = {}_{20}C_{19} (0.91)^{19} (0.09)^1 = 0.29996$$

$$P(x=20) = {}_{20}C_{20} (0.91)^{20} (0.09)^0 = 0.15164$$

2

$$P(x \le 17) = 1 - P(x=18) - P(x=19) - P(x=20)$$

$$= 0.26657$$

or $\boxed{27\%}$

1

---

note: if calculated

$$P(x=17) = {}_{20}C_{17} (0.91)^{17} (0.09)^3$$

$$= 0.167238$$

$\boxed{= 16.7\%}$ as final answer

-2

3. (5 points) Your engineering company is considering competing for Project Alpha. The cost of competing for Project Alpha is \$35 000. You estimate that your bid has a 45% probability of success, which will mean a profit of \$180 000.

(a) Calculate the expected earnings, where earnings = profit − cost

x = earnings*	$p(x)$
−35	0.55
180−35 = 145	0.45

1

* in thousands of dollars

$$E(x) = \sum x\,p(x)$$
$$= (-35)(0.55)$$
$$(145)(0.45)$$
$$= 46$$

1

so $\boxed{\$46\,000}$

(b) Your company does not like to bid on risky projects in which the standard deviation of the earnings is more than \$60 000. Should your firm bid on Project Alpha? Explain your reasoning.

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$
$$= (-35)^2(0.55) + (145)^2(0.45) - (46)^2$$
$$= 8019$$

1

$$\sigma = \sqrt{\sigma} = 89.5489$$

so $\boxed{\sigma = \$89\,549}$

1

no, the firm should not bid on this contract since σ is greater than the guideline

1

4. (4 points) Suppose that some phenomenon has the following probability distribution.

$$f(x) = \begin{cases} \dfrac{k}{x} & \text{for } 1 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate k so that $f(x)$ is indeed a probability distribution function. Give both an exact and an approximate answer.

$$1 = \int_{-\infty}^{\infty} f(x)\,dx$$

$$1 = \int_1^{10} \frac{k}{x}\,dx$$

$$1 = k \ln x \Big|_1^{10}$$

$$1 = k(\ln 10 - \cancelto{0}{\ln 1})$$

$$1 = k \cdot \ln 10$$

so $\boxed{k = \dfrac{1}{\ln 10} \approx 0.434}$

(b) Calculate the mean value of x. Give both an exact and an approximate answer.

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

$$= \int_1^{10} x \cdot \frac{k}{x}\,dx$$

$$= \int_1^{10} k\,dx$$

$$= kx \Big|_1^{10}$$

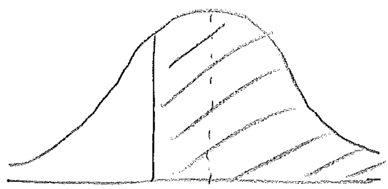
$$= k(10-1)$$

$\mu = 9k$

$\boxed{= \dfrac{9}{\ln 10} \approx 3.909}$

5. (6 points) The mean price for a barrel of crude oil in July 2014 was \$105. Let's assume that the price is normally distributed with a standard deviation of \$8.

(a) Find the probability that the price for a barrel of crude oil is above \$100.



$z = -0.63$

$$z = \frac{x - \mu}{\sigma} = \frac{100 - 105}{8}$$

$$= -0.625$$

$$= -0.63 \quad (1)$$

area = 0.2357 (1)

(0.2324 for $z = -0.62$ or an interpolated value also acceptable)

$$p = 0.5 + 0.2357 \quad (1)$$

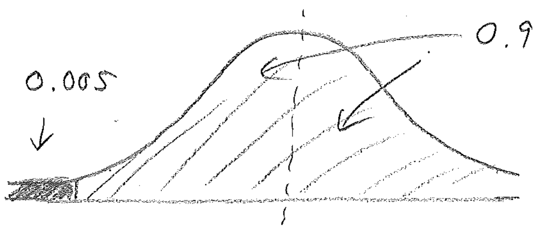
$$= 0.7357 \quad (\text{or } 0.7324)$$

$$= 73.6\%$$

(b) 99.5% of the time, the price is above a certain amount. Calculate that amount.

0.995 in total

0.005



(1)

area 0.495 has $z = -2.575$ (1)

(-2.57 or -2.58 also acceptable)

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma \quad (1)$$

$$= 105 + (-2.575)(8)$$

$$= 84.4$$

so $\boxed{\$84.4}$

(\$84.44 and \$84.36 also acceptable)