4 Discrete Random Variables

1. a) $P(10 \le X \le 15) = P(10.89) + P(12.31) + P(14.22)$
$= 0.28 + 0.13 + 0.11$
$= 0.52$

b) $\mu = E(X) = \sum x P(x)$
$= 9.12(0.41) + 10.89(0.28) + 12.31(0.13) + 14.22(0.11) + 15.06(0.07)$
$= 11.007$

c) $\sigma^2 = E(X^2) - \mu^2 = 125.126 - 11.007^2 = 3.972$

$E(X^2) = \sum x^2 P(x)$
$= 9.12^2(0.41) + 10.89^2(0.28) + 12.31^2(0.13) + 14.22^2(0.11) + 15.06^2(0.07)$
$= 125.126$

d) $\sigma = \sqrt{3.972} = 1.993$

e) $P(\mu - 2\sigma \le X \le \mu + 2\sigma) = P(7.021 \le X \le 14.993)$
$= P(9.12) + P(10.89) + P(12.31) + P(14.22)$
$= 0.41 + 0.28 + 0.13 + 0.11$
$= 0.93$

2. a) $P(Y < 2) = P(-4) + P(-3)$
$= 0.4 + 0.2$
$= 0.6$

b) $\mu = E(Y) = \sum y P(y)$
$= (-4)(0.4) + (-3)(0.2) + 2(0.1) + 7(0.3)$
$= 0.1$

c) $\sigma^2 = E(Y^2) - \mu^2 = 23.3 - 0.1^2 = 23.29$

$$E(Y^2) = \sum y^2 P(y)$$
$$= (-4)^2(0.4) + (-3)^2(0.2) + 2^2(0.1) + 7^2(0.3)$$
$$= 23.3$$

d) $\sigma = \sqrt{23.29} = 4.83$

e) $P(\mu - \sigma \le X \le \mu + \sigma) = P(-4.73 \le X \le 4.93)$
$$= P(-4) + P(-3) + P(2)$$
$$= 0.4 + 0.2 + 0.1$$
$$= 0.7$$

3. let X = earnings of Project A (\$)
Y = earning of Project B (\$)

x	P(x)
10000	0.6
5000	0.3
0	0.1

y	P(y)
25000	0.8
0	0.2

a) $E(X) = \sum x P(x) = 10000(0.6) + 5000(0.3) + 0(0.1)$
$= \$7500$

b) $E(Y) = \sum y P(y) = 25000(0.8) + 0(0.2)$
$= \$20000$

c) $\sigma_X^2 = E(X^2) - \mu^2 = 67500000 - 7500^2 = 11{,}250{,}000 \quad \$^2$

$E(X^2) = \sum x^2 P(x) = 10000^2(0.6) + 5000^2(0.3) + 0^2(0.1)$
$= 67{,}500{,}000$

$\mu = E(X)$

d) $\sigma_Y^2 = E(Y^2) - \mu^2 = 500000000 - 20000^2 = 100{,}000{,}000 \quad \$^2$

$E(Y^2) = \sum y^2 P(y) = 25000^2(0.8) + 0^2(0.2)$
$= 500{,}000{,}000$

$\mu = E(Y)$

e) Project B, since $\sigma_Y^2 > \sigma_X^2$

4. a) let X = earnings = revenue − cost ($)

success $\Rightarrow$ $x = 80000 - 10000 = 70000$ $\quad P(\text{success}) = 0.35$
failure $\Rightarrow$ $x = 0 - 10000 = -10000$ $\quad P(\text{failure}) = 1 - 0.35 = 0.65$

x	P(x)
70000	0.35
-10000	0.65

b) $E(X) = \sum x P(x) = 70000(0.35) + (-10000)(0.65)$
$= \$18000$

c) $\sigma = \sqrt{\sigma^2} \approx \$38,000$

$\sigma^2 = E(X^2) - \mu^2 = 1780000000 - 18000^2 = 1,456,000,000$

$E(X^2) = 70000^2(0.35) + (-10000)^2(0.65)$
$= 1,780,000,000$

$\mu = E(X)$

d) Project Alpha, since $\sigma_x > 25000$

5. a) let X = insurance company's gain ($)

theft $\Rightarrow$ $x = m - 4000$

no theft $\Rightarrow$ $x = m$

P(theft) = 0.013

P(no theft) = 1 - 0.013 = 0.987

x	P(x)
m - 4000	0.013
m	0.987

b) $E(X) = 60$

$60 = \sum x P(x)$

$60 = (m - 4000)(0.013) + m(0.987)$

$60 = m - 52$

$m = \$112$