5 Binomial and Poisson Distributions

1. this is the set-up for the Binomial distribution with
X = number of successful free throws
$n = 6 \quad p = 0.72 \quad q = 1 - 0.72 = 0.28$

a) $P(X=5) = 6C5 \cdot (0.72)^5 (0.28)^1 = 0.33$

b) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$
$= 6C4 \cdot (0.72)^4 (0.28)^2 + 6C5 \cdot (0.72)^5 (0.28)^1 + 6C6 \cdot (0.72)^6 (0.28)^0$
$= 0.78$

c) $P(X \geq 2) = 1 - P(X<2)$
$= 1 - [P(X=0) + P(X=1)]$
$= 1 - 6C0 \cdot (0.72)^0 (0.28)^6 - 6C1 \cdot (0.72)^1 (0.28)^5$
$= 0.99$

2. this is the set-up for the Binomial distribution with
X = number of correct answers
$n = 20 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$

a) $P(X=6) = 20C6 \cdot \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{14} = 0.18$

b) $P(5 \leq X \leq 7) = P(X=5) + P(X=6) + P(X=7)$
$= 20C5 \cdot \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{15} + 20C6 \cdot \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{14} + 20C7 \cdot \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^{13}$
$= 0.51$

c) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= 20C0 \cdot \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20} + 20C1 \cdot \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{19} + 20C2 \cdot \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18} + 20C3 \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{17}$
$= 0.06$

3. this is the set-up for the Poisson distribution with
X = number of cracks per m^3
$\lambda = 1.7$

a) $P(X \geq 1) = 1 - P(X=0)$
$= 1 - \frac{1.7^0 e^{-1.7}}{0!}$
$= 0.82$

b) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= \frac{1.7^0 e^{-1.7}}{0!} + \frac{1.7^1 e^{-1.7}}{1!} + \frac{1.7^2 e^{-1.7}}{2!} + \frac{1.7^3 e^{-1.7}}{3!}$
$= 0.91$

4. this is the set-up for the Poisson distribution with
X = number of typos per page
$\lambda = \frac{400}{1000} = 0.4$

a) $P(X=2) = \frac{0.4^2 e^{-0.4}}{2!} = 0.05$

b) $P(X>1) = 1 - P(X \leq 1)$
$= 1 - [P(X=0) + P(X=1)]$
$= 1 - \frac{0.4^0 e^{-0.4}}{0!} - \frac{0.4^1 e^{-0.4}}{1!}$
$= 0.06$

5. this is the set-up for a Poisson distribution with

X = number of requests per hour

$$\lambda = \frac{3 \text{ requests}}{\frac{1}{4} \text{ hour}} = 12 \frac{\text{requests}}{\text{hour}}$$

$$P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{12^0 e^{-12}}{0!} + \frac{12^1 e^{-12}}{1!} + \frac{12^2 e^{-12}}{2!} + \frac{12^3 e^{-12}}{3!} + \frac{12^4 e^{-12}}{4!}$$

$$= 0.008$$