8 Central Limit Theorem

1. we want $P(6 < \bar{x} < 8)$

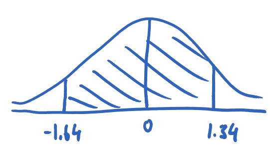
$\mu = 7.1$, $\sigma = 5.2$, $n = 60$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\bar{x} = 6 \Rightarrow z = \dfrac{6 - 7.1}{5.2/\sqrt{60}} = -1.64$

$\bar{x} = 8 \Rightarrow z = \dfrac{8 - 7.1}{5.2/\sqrt{60}} = 1.34$

$P(6 < \bar{x} < 8) = P(-1.64 < z < 1.34)$

$=$ 

$= 0.4495 + 0.4099$
$= 0.8594$

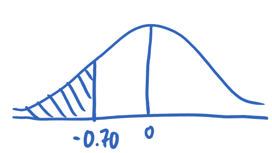
2. we want $P(\bar{x} < 70)$

$\mu = 71$, $\sigma = 9$

a) $n = 40$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\bar{x} = 70 \Rightarrow z = \dfrac{70 - 71}{9/\sqrt{40}} = -0.70$

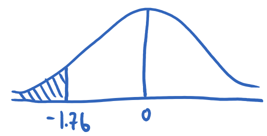
$P(\bar{x} < 70) = P(z < -0.70) =$ 

$= 0.5 - 0.2580 = 0.2420$

b) $n = 250$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ $\quad \bar{x} = 70 \Rightarrow z = \dfrac{70 - 71}{9/\sqrt{250}} = -1.76$

$P(\bar{x} < 70) = P(z < -1.76) =$



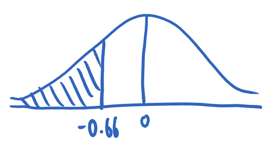
$= 0.5 - 0.4608 = 0.0392$

3. $\mu = 513.3 \quad \sigma = 31.5 \quad n = 40$

a) we want $P(\bar{x} < 510)$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ $\quad \bar{x} = 510 \Rightarrow z = \dfrac{510 - 513.3}{31.5/\sqrt{40}} = -0.66$

$P(\bar{x} < 510) = P(z < -0.66) =$

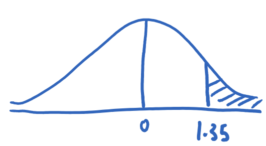


$= 0.5 - 0.2454 = 0.2546$

b) we want $P(\bar{x} > 520)$

$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ $\quad \bar{x} = 520 \Rightarrow z = \dfrac{520 - 513.3}{31.5/\sqrt{40}} = 1.35$

$P(\bar{x} > 520) = P(z > 1.35) =$



$= 0.5 - 0.4115 = 0.0885$

c) $P(510 < \bar{x} < 520) = 1 - [P(\bar{x} < 510) + P(\bar{x} > 520)]$

$= 1 - [0.2546 + 0.0885]$

$= 0.6569$

4. we want $P(\text{total} > 7344)$

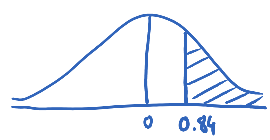
$\mu = 45.5 \qquad \sigma = 6 \qquad n = 160$

$\bar{x} = \dfrac{\text{total}}{n} \qquad \text{total} = 7344 \Rightarrow \bar{x} = \dfrac{7344}{160} = 45.9$

$P(\text{total} > 7344) = P(\bar{x} > 45.9)$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \bar{x} = 45.9 \Rightarrow z = \dfrac{45.9 - 45.5}{6/\sqrt{160}} = 0.84$

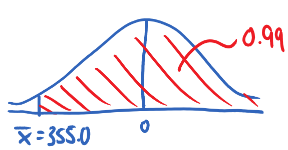
$P(\bar{x} > 45.9) = P(z > 0.84) =$



$= 0.5 - 0.2995 = 0.2005$

5. we want μ such that $P(\bar{x} > 355.0) = 0.99$

$\sigma = 1.9 \qquad n = 30$



reverse look-up:

area $= 0.99 - 0.5 = 0.49 \Rightarrow z = 2.33$

but $z < 0$ so $z = -2.33$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$-2.33 = \dfrac{355.0 - \mu}{1.9/\sqrt{30}}$

$-0.8 = 355.0 - \mu$

$\mu = 355.8$ mL

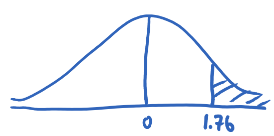
6. we want $P(\text{total} > 6000)$

$\mu = 145$ $\sigma = 18$ $n = 40$

$\bar{x} = \frac{\text{total}}{n}$ $\text{total} = 6000 \Rightarrow \bar{x} = \frac{6000}{40} = 150$

$P(\text{total} > 6000) = P(\bar{x} > 150)$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ $\bar{x} = 150 \Rightarrow z = \frac{150 - 145}{18/\sqrt{40}} = 1.76$

$P(\bar{x} > 150) = P(z > 1.76) =$ 

$= 0.5 - 0.4608 = 0.0392$