9 Confidence Intervals

1. $n = 40$ $\quad \bar{x} = 78.0$ $\quad s = 5.0$ $\quad \sigma \approx s$ $\quad C = 0.95$

$$\Downarrow$$

$$z_{\alpha/2} = 1.960$$

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 78 \pm 1.96 \left(\frac{5}{\sqrt{40}} \right) = 78 \pm 1.5$$

$$76.5 < \mu < 79.5$$

2. $n = 50$ $\quad \bar{x} = 28.00$ $\quad s = 3.32$ $\quad \sigma \approx s$

a) $C = 0.95 \Rightarrow z_\alpha = 1.645$

$$\bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} = 28 + 1.645 \left(\frac{3.32}{\sqrt{50}} \right) = 28.77$$

$$\mu < 28.77\ ^\circ C$$

b) $C = 0.98 \Rightarrow z_\alpha = 2.054$

$$\bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} = 28 + 2.054 \left(\frac{3.32}{\sqrt{50}} \right) = 28.96$$

$$\mu < 28.96\ ^\circ C$$

3. $\sigma = 0.08 \qquad n = 100 \qquad \bar{x} = 11.00$

a) $C = 0.9 \Rightarrow z_\alpha = 1.282$

$$LCB = \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} = 11 - 1.282\left(\frac{0.08}{\sqrt{100}}\right) = 10.99$$

$\mu > 10.99$ inches

b) $C = 0.99 \Rightarrow z_\alpha = 2.326$

$$LCB = \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} = 11 - 2.326\left(\frac{0.08}{\sqrt{100}}\right) = 10.98$$

$\mu > 10.98$ inches

4. we want n such that $ME \leq 2.0$

$C = 0.99 \Rightarrow z_{\alpha/2} = 2.576 \qquad \sigma = 5.9$

$$ME \leq 2.0$$
$$z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq 2.0$$
$$2.576\left(\frac{5.9}{\sqrt{n}}\right) \leq 2.0$$
$$7.5992 \leq \sqrt{n}$$
$$n \geq 57.7$$

(round up to the nearest integer)

so 58 is the minimum sample size

5. we want n such that $ME < 2$

$C = 0.95 \Rightarrow z_{\alpha/2} = 1.960 \qquad \sigma = 13$

$$ME < 2$$
$$z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < 2$$
$$1.96\left(\frac{13}{\sqrt{n}}\right) < 2$$
$$12.74 < \sqrt{n}$$
$$n > 162.3$$

(round up to the nearest integer)

so 163 is the minimum sample size

6. $ME = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

a) ME decreases
b) ME increases
c) C increases $\Rightarrow z_{\alpha/2}$ increases $\Rightarrow$ ME increases
d) no change to ME

7. $n = 10 \qquad \bar{x} = 25.0 \qquad s = 5.1$

$C = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$
$df = n - 1 = 10 - 1 = 9$
$\Big\} \; t_{\alpha/2} = 2.262$

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 25 \pm 2.262\left(\frac{5.1}{\sqrt{10}}\right) = 25 \pm 3.6$$

$$21.4 < \mu < 28.6 \text{ ppm}$$

8. $n = 15 \qquad \bar{x} = 69.1 \qquad s = 3.5$

$C = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005 \Rightarrow t_{\alpha/2} = 2.977$

$df = n - 1 = 15 - 1 = 14$

$$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 69.1 \pm 2.977\left(\frac{3.5}{\sqrt{15}}\right) = 69.1 \pm 2.7$$

$$66.4 < \mu < 71.8 \text{ pounds}$$

9. $n = 12 \qquad \bar{x} = 51 \qquad s = 2$

a) $C = 0.9 \Rightarrow \alpha = 0.1 \Rightarrow t_\alpha = 1.363$

$df = n - 1 = 12 - 1 = 11$

$$UCB = \bar{x} + t_\alpha \frac{s}{\sqrt{n}} = 51 + 1.363\left(\frac{2}{\sqrt{12}}\right) = 51.8$$

$$\mu < 51.8 \text{ miles per gallon}$$

b) $C = 0.975 \Rightarrow \alpha = 0.025 \Rightarrow t_\alpha = 2.201$

$df = n - 1 = 12 - 1 = 11$

$$UCB = \bar{x} + t_\alpha \frac{s}{\sqrt{n}} = 51 + 2.201\left(\frac{2}{\sqrt{12}}\right) = 52.3$$

$$\mu < 52.3 \text{ miles per gallon}$$

10. $n = 20$ $\quad \bar{x} = 356.1$ $\quad s = 1.9$

a) $C = 0.95 \Rightarrow \alpha = 0.05$
$df = n - 1 = 20 - 1 = 19$
$\Big\} \; t_\alpha = 1.729$

$$LCB = \bar{x} - t_\alpha \frac{s}{\sqrt{n}} = 356.1 - 1.729\left(\frac{1.9}{\sqrt{20}}\right) = 355.4$$

$$\mu > 355.4 \text{ mL}$$

b) $C = 0.99 \Rightarrow \alpha = 0.01$
$df = 19$
$\Big\} \; t_\alpha = 2.539$

$$LCB = \bar{x} - t_\alpha \frac{s}{\sqrt{n}} = 356.1 - 2.539\left(\frac{1.9}{\sqrt{20}}\right) = 355.0$$

$$\mu > 355.0 \text{ mL}$$

11. $n = 6$ $\quad \bar{x} = 232.26$ $\quad s = 0.14$

$C = 0.98 \Rightarrow \alpha = 0.02 \Rightarrow \alpha/2 = 0.01$
$df = n - 1 = 6 - 1 = 5$
$\Big\} \; t_{\alpha/2} = 3.365$

$$ME = t_{\alpha/2} \frac{s}{\sqrt{n}} = 3.365\left(\frac{0.14}{\sqrt{6}}\right) = 0.19 \text{ °C}$$