Complex Numbers Exercises

Answers appear on the next page.

1. Express $z = (1 + 2i)(4 - 6i)^2$ in the form $a + bi$.

2. Find $\overline{z}$ and $|z|$ for:
a) $z = 2 + 7i$
b) $z = -3 - 5i$

3. Find $\frac{1}{z}$ for:
a) $z = 1 - 5i$
b) $z = 1 + i$

4. Find $\frac{z_1}{z_2}$ for $z_1 = 2 + i$ and $z_2 = -7 + 5i$.

5. Let $z_1 = 2 + 2\sqrt{3}i$ and $z_2 = \frac{\sqrt{3}}{3} + \frac{1}{3}i$.
a) Convert z_1 and z_2 to polar form.
b) Compute $z_1 z_2$
c) Compute $\frac{z_1}{z_2}$.

6. Compute $(i + 1)^8$ by converting $i + 1$ to polar form.

7. Solve $z^6 - 64 = 0$.

8. Let $w = a + bi$ and $z = c + di$. Show that $|wz| = |w||z|$.

ANSWERS

1. $z = 76 - 88i$

2. a) $2 - 7i$, $\sqrt{53}$
b) $-3 + 5i$, $\sqrt{34}$

3. a) $\frac{1}{26} + \frac{5}{26}i$
b) $\frac{1}{2} - \frac{1}{2}i$

4. $\frac{-9}{74} - \frac{17}{74}i$.

5. a) $z_1 = 4[\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})]$ and $z_2 = \frac{2}{3}[\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})]$.
b) $\frac{8}{3}[\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})] = \frac{8}{3}i$.
c) $6[\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})]$.

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7. $z = \pm 2, 1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$.

8.

$$\begin{aligned}
|wz| &= |(ac - bd) + (ad + bc)i| \\
&= \sqrt{(ac - bd)^2 + (ad + bc)^2} \\
&= \sqrt{[(ac)^2 - 2acbd + (bd)^2] + [(ad)^2 + 2adbc + (bc)^2]} \\
&= \sqrt{(ac)^2 + (bd)^2 + (ad)^2 + (bc)^2} \\
&= \sqrt{(a^2 + b^2)c^2 + (a^2 + b^2)d^2} \\
&= \sqrt{(a^2 + b^2)(c^2 + d^2)} \\
&= \sqrt{a^2 + b^2}\sqrt{c^2 + d^2} \\
&= |w||z|
\end{aligned}$$