

(14) Write $\vec{v} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 13 \\ 7 & 2 & 2 & -6 \end{array} \right] \end{array}$$

$$R_2 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 7 & 2 & 2 & -6 \\ 0 & 0 & 1 & 13 \end{array} \right]$$

$$R_2 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 2 & 2 & -90 \\ 0 & 0 & 1 & 13 \end{array} \right]$$

$$R_2/2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & -45 \\ 0 & 0 & 1 & 13 \end{array} \right]$$

$$R_2 - R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -58 \\ 0 & 0 & 1 & 13 \end{array} \right]$$

$$\vec{v} = 12\vec{v}_1 - 58\vec{v}_2 + 13\vec{v}_3$$

$$T(\vec{v}) = T(12\vec{v}_1 - 58\vec{v}_2 + 13\vec{v}_3)$$

$$= 12T(\vec{v}_1) - 58T(\vec{v}_2) + 13T(\vec{v}_3)$$

because T is linear

$$= 12 \begin{bmatrix} -6 \\ 30 \\ 24 \\ 8 \end{bmatrix} - 58 \begin{bmatrix} -2 \\ 8 \\ 6 \\ 2 \end{bmatrix} + 13 \begin{bmatrix} -1 \\ 9 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 31 \\ 13 \\ 18 \\ -20 \end{bmatrix}$$

(15)

$$|A| = \begin{vmatrix} 2 & 1 & 6 & 9 \\ 2 & 1 & 3 & 0 \\ 6 & 8 & 1 & 0 \\ 4 & 12 & 1 & 1 \end{vmatrix}$$

Get an upper triangular determinant.

$$\begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \\ R_4 - 2R_1 \end{array} = \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 0 & -3 & -9 \\ 0 & 5 & -17 & -27 \\ 0 & 10 & -11 & -17 \end{vmatrix}$$

$$R_2 \leftrightarrow R_3 = - \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & -3 & -9 \\ 0 & 10 & -11 & -17 \end{vmatrix}$$

$$R_4 - 2R_2 = - \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 23 & 37 \end{vmatrix}$$

$$R_3 / (-3) = 3 \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 23 & 37 \end{vmatrix}$$

$$R_4 - 23R_3 = 3 \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -32 \end{vmatrix}$$

upper triangular ✓

$$\begin{aligned} \therefore |A| &= 3(2 \cdot 5 \cdot 1 \cdot (-32)) \\ &= -960 \end{aligned}$$

(16)

$$\text{Matrix of cofactors} = \begin{bmatrix} 0 & 2 & -1 \\ -21 & 5 & 1 \\ 7 & -2 & 1 \end{bmatrix} \quad \rightarrow \text{transpose}$$

$$\text{adj}(A) = \begin{bmatrix} 0 & -21 & 7 \\ 2 & 5 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \frac{1}{7} \begin{bmatrix} 0 & -21 & 7 \\ 2 & 5 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

expanding along col. 1

$$|A| = 1 \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} + 1 \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= 1(0) + 1(7)$$
$$= 7$$

(17) a) $|\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 & -1 & -6 \\ 0 & \lambda - 4 & -2 & 0 \\ 0 & 0 & \lambda - 2 & -8 \\ 0 & 0 & 0 & \lambda - 2 \end{vmatrix} \leftarrow \text{upper triangular}$

$$= (\lambda - 2)^3 (\lambda - 4)$$

b) $[A - 2I | \vec{0}]$

$$\begin{bmatrix} 0 & 1 & 1 & 6 & | & 0 \\ 0 & 2 & 2 & 0 & | & 0 \\ 0 & 0 & 0 & 8 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 0 & 1 & 1 & 6 & | & 0 \\ 0 & 0 & 0 & -12 & | & 0 \\ 0 & 0 & 0 & 8 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$R_2 / (-12)$

$$\begin{bmatrix} 0 & 1 & 1 & 6 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & 8 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$R_1 - 6R_2$
 $R_3 - 8R_2$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

\uparrow \uparrow $x_2 + x_3 = 0$
 $x_1 = s$ $x_3 = t$ $x_2 = -t$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} t$$

Basis = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

$x_4 = 0$

c) The algebraic multiplicity of
 $\lambda=2$ is 3 [from $(\lambda-2)^3$]

but the geometric multiplicity of
 $\lambda=2$ is only 2 [E_2 has 2 basis vectors].

Need alg. mult = geo. mult for
each eigenvalue in order for A to
be diagonalizable.

$$(18) \quad |\lambda I - A| = \begin{vmatrix} \lambda - 1 & 0 & -2 \\ 0 & \lambda & -6 \\ 0 & 0 & \lambda + 3 \end{vmatrix}$$

$$= (\lambda - 1)\lambda(\lambda + 3)$$

Eigenvalues are $\lambda = 1, 0, -3$

$$E_1: [A - I | \vec{0}]$$

$$\begin{bmatrix} 0 & 0 & 2 & | & 0 \\ 0 & -1 & 6 & | & 0 \\ 0 & 0 & -4 & | & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{bmatrix} 0 & -1 & 6 & | & 0 \\ 0 & 0 & 2 & | & 0 \\ 0 & 0 & -4 & | & 0 \end{bmatrix}$$

$$\begin{matrix} R_1/(-1) \\ R_2/2 \end{matrix} \begin{bmatrix} 0 & 1 & -6 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & -4 & | & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 + 6R_2 \\ R_3 + 4R_2 \end{matrix} \begin{bmatrix} 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{matrix} \uparrow \\ x_1 = t \end{matrix} \quad \begin{matrix} x_2 = 0 \\ x_3 = 0 \end{matrix}$$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t$$

$$E_1 = \text{span} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$E_0: [A - 0I | \vec{0}]$$

$$\begin{bmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 0 & 6 & | & 0 \\ 0 & 0 & -3 & | & 0 \end{bmatrix}$$

$$R_2/6 \begin{bmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & -3 & | & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 - 2R_2 \\ R_3 + 3R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{matrix} \uparrow \\ x_2 = t \\ x_1 = 0 \\ x_3 = 0 \end{matrix}$$

$$\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t$$

$$E_0 = \text{span} \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

$$E_{-3}: [A+3I | \vec{0}]$$

$$\left[\begin{array}{ccc|c} 4 & 0 & 2 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1/4 \\ R_2/3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\boxed{x_3 = t}$$

$$x_2 + 2x_3 = 0$$

$$\boxed{x_2 = -2t}$$

$$x_1 + \frac{1}{2}x_3 = 0$$

$$\boxed{x_1 = -\frac{t}{2}}$$

$$\vec{x} = \begin{bmatrix} -\frac{1}{2} \\ -2 \\ 1 \end{bmatrix} t$$

$$\text{or } \vec{x} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} t$$

$$E_{-3} = \text{span} \left(\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} \right)$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{or } P = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{or } P = \begin{bmatrix} 0 & 1 & -\frac{1}{2} \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \text{ etc.}$$

(19) Since \mathcal{B} is an orthogonal basis

$$\{ \vec{b}_1, \vec{b}_2, \vec{b}_3 \}$$

$$\begin{aligned} \vec{u} &= \frac{\vec{u} \cdot \vec{b}_1}{\|\vec{b}_1\|^2} \vec{b}_1 + \frac{\vec{u} \cdot \vec{b}_2}{\|\vec{b}_2\|^2} \vec{b}_2 + \frac{\vec{u} \cdot \vec{b}_3}{\|\vec{b}_3\|^2} \vec{b}_3 \\ &= \frac{-150}{75} \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix} + \frac{342}{114} \begin{bmatrix} 7 \\ 7 \\ 4 \end{bmatrix} + \frac{2850}{8550} \begin{bmatrix} -69 \\ 45 \\ 42 \end{bmatrix} \end{aligned}$$

\nearrow
 $\left\| \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix} \right\|^2 = 75$

$$= -2 \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix} + 3 \begin{bmatrix} 7 \\ 7 \\ 4 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} -69 \\ 45 \\ 42 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 46 \\ 12 \end{bmatrix}$$

$$\textcircled{20.} \quad W = \left\{ \begin{bmatrix} 5t \\ -2t \\ 10t \end{bmatrix} \right\} \\ = \text{span} \left(\begin{bmatrix} 5 \\ -2 \\ 10 \end{bmatrix} \right)$$

Basis for $W^\perp =$ Basis for $\text{null}(A)$

where $A = \begin{bmatrix} 5 & -2 & 10 \end{bmatrix}$

Solve $\begin{bmatrix} 5 & -2 & 10 & | & 0 \end{bmatrix}$

$R_1/5 \quad \begin{bmatrix} 1 & -\frac{2}{5} & 2 & | & 0 \end{bmatrix}$

$\begin{matrix} \uparrow & \uparrow \\ x_2 = s & x_3 = t \end{matrix}$

$$x_1 - \frac{2}{5}x_2 + 2x_3 = 0$$

$$x_1 = \frac{2}{5}s - 2t$$

$$\vec{x} = \begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} t$$

Basis for $W^\perp = \left\{ \begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$(21) \text{ a) } \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad X = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix} - \text{proj}_X \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix} - \frac{6}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} \quad X = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} \right\}$$

$$\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - \text{proj}_X \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{12}{44} \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix}$$

Can scale to simplify arithmetic:

$$11\vec{v}_3 = 11 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - 11 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 4 \\ 33 \end{bmatrix}$$

$\{\vec{v}_1, \vec{v}_2, 11\vec{v}_3\}$ is an orthogonal basis.

Orthonormal basis:

$$= \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{11} \\ 1/\sqrt{11} \\ 3/\sqrt{11} \\ 0 \end{bmatrix}, \begin{bmatrix} 6/\sqrt{1177} \\ -6/\sqrt{1177} \\ 4/\sqrt{1177} \\ 33/\sqrt{1177} \end{bmatrix} \right\}$$

$$b) \quad Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{11} & 6/\sqrt{1177} \\ 1/\sqrt{2} & 1/\sqrt{11} & -6/\sqrt{1177} \\ 0 & 3/\sqrt{11} & 4/\sqrt{1177} \\ 0 & 0 & 33/\sqrt{1177} \end{bmatrix}$$

and if $A = QR$

$$Q^T A = Q^T Q R \quad [Q^T Q = I]$$

$$Q^T A = R$$

$$R = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{11} & 1/\sqrt{11} & 3/\sqrt{11} & 0 \\ 6/\sqrt{1177} & -6/\sqrt{1177} & 4/\sqrt{1177} & 33/\sqrt{1177} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{2} & 6/\sqrt{2} & 2/\sqrt{2} \\ 0 & 22/\sqrt{11} & 6/\sqrt{11} \\ 0 & 0 & 107/\sqrt{1177} \end{bmatrix}$$

$$\text{Check: } A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{11}} & \frac{6}{\sqrt{1177}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{11}} & \frac{-6}{\sqrt{1177}} \\ 0 & \frac{3}{\sqrt{11}} & \frac{4}{\sqrt{1177}} \\ 0 & 0 & \frac{33}{\sqrt{1177}} \end{bmatrix} \begin{bmatrix} 2/\sqrt{2} & 6/\sqrt{2} & 2/\sqrt{2} \\ 0 & 22/\sqrt{11} & 6/\sqrt{11} \\ 0 & 0 & 107/\sqrt{1177} \end{bmatrix} \checkmark$$

(22) Eigenvalues of A are $\lambda = 2, 5$

$E_2: [A - 2I | \vec{0}]$

$$\begin{bmatrix} 2 & -1 & -1 & | & 0 \\ -1 & 2 & -1 & | & 0 \\ -1 & -1 & 2 & | & 0 \end{bmatrix} \xrightarrow{R_1+2R_2} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{bmatrix} -1 & 2 & -1 & | & 0 \\ 2 & -1 & -1 & | & 0 \\ -1 & -1 & 2 & | & 0 \end{bmatrix}$$

$$R_1 / (-1) \begin{bmatrix} 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \\ -1 & -1 & 2 & | & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 + R_1 \end{array} \begin{bmatrix} 1 & -2 & 1 & | & 0 \\ 0 & 3 & -3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{bmatrix}$$

$$R_2/3 \begin{bmatrix} 1 & -2 & 1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{bmatrix}$$

$$\begin{array}{l} \uparrow \\ \boxed{x_3 = t} \\ x_1 - x_3 = 0 \quad \boxed{x_1 = t} \\ x_2 - x_3 = 0 \quad \boxed{x_2 = t} \end{array}$$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

Orthonormal basis
for E_2 : $\left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \right\}$

$$E_5: [A - sI | \vec{0}]$$

$$\begin{bmatrix} -1 & -1 & -1 & | & 0 \\ -1 & -1 & -1 & | & 0 \\ -1 & -1 & -1 & | & 0 \end{bmatrix}$$

$$\begin{array}{l} R_1 / (-1) \\ R_2 - R_1 \\ R_3 - R_1 \end{array} \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{array}{c} \uparrow \\ x_2 = s \\ \uparrow \\ x_3 = t \end{array}$$

$$x_1 + x_2 + x_3 = 0 \quad \boxed{x_1 = -s - t}$$

$$\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t$$

$$\text{Basis for } E_5 = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Orthonormal basis:

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad X = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$\begin{aligned} \vec{v}_2 &= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \text{proj}_X \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 2\vec{v}_2 &= 2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \end{aligned}$$

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

Orthonormal basis:

$$\left\{ \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix} \right\}$$

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$$

(23)

$$z = \frac{x + yi}{13 + 5i}$$

$$= \frac{(x + yi) \cdot (13 - 5i)}{(13 + 5i)(13 - 5i)}$$

$$= \frac{13x + 5y + (13y - 5x)i}{169 + 25}$$

$$= \frac{13x + 5y}{194} + \frac{13y - 5x}{194} i$$

(24)

$$z = -8i \quad |z| \text{ must be } \geq 0$$

$$= 8[0 - i]$$

$$\left. \begin{array}{l} \cos \theta = 0 \\ \sin \theta = -1 \end{array} \right\} \theta = \frac{3\pi}{2}$$

$$= 8 \left[\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right]$$

$$z^{1/3} = 8^{1/3} \left[\cos \frac{\frac{3\pi}{2} + 2\pi\alpha}{3} + i \sin \frac{\frac{3\pi}{2} + 2\pi\alpha}{3} \right]$$

$$\alpha = 0, 1, 2$$

α	$\frac{3\pi + 4\pi\alpha}{6}$
0	$\frac{\pi}{2}$
1	$\frac{7\pi}{6}$
2	$\frac{11\pi}{6}$

$$z_1^{1/3} = 2 \left[\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right] = 2i$$

$$z_2^{1/3} = 2 \left[\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right] = 2 \left[-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right] = -\sqrt{3} - i$$

$$z_3^{1/3} = 2 \left[\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right] = 2 \left[\frac{\sqrt{3}}{2} - \frac{1}{2}i \right] = \sqrt{3} - i$$