Math173 – Supplement to Section 7.3

1. Change the following complex numbers to the form $re^{i\theta}$.

   a) -7
   b) $6i$
   c) $4 - 3i$
   d) $\sqrt{2} + i\sqrt{2}$
   e) $12\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

2. Change the following complex numbers to the form $a+bi$.

   a) $5e^{i\pi}$
   b) $12e^{-i\frac{\pi}{3}}$
   c) $2e^{i\frac{\pi}{2}}$

3. Multiply or divide the following complex numbers.

   a) $\left(5e^{i\pi}\right)\left(3e^{i\pi}\right)$
   b) $\left(12e^{-i\frac{\pi}{3}}\right)\left(4e^{i\frac{\pi}{2}}\right)$
   c) $\left(\sqrt{2} + i\sqrt{2}\right)\left(1 - i\sqrt{3}\right)$
   d) $\left(12e^{-i\frac{\pi}{3}}\right) \div \left(4e^{i\frac{\pi}{2}}\right)$
   e) $\dfrac{\left(\sqrt{2} + i\sqrt{2}\right)}{\left(1 - i\sqrt{3}\right)}$

4. Raise the following complex numbers to the given power and simplify. Write your answer in the form $a+bi$.

   a) $\left(5e^{i\pi}\right)^4$
   b) $\left(12e^{-i\frac{\pi}{3}}\right)^9$
   c) $\left(2e^{i\frac{\pi}{2}}\right)^7$
   d) $\left(\sqrt{2} + i\sqrt{2}\right)^8$

5. Find the fourth roots of the following numbers. Do so by changing them into the form $re^{i\theta}$ and using the laws of exponents and DeMoivre's theorem. Leave your answer in the form $re^{i\theta}$.

   a) i
   b) $\left(\sqrt{2}+i\sqrt{2}\right)$
   c) –16
   d) $1-i\sqrt{3}$

Solutions

1. a) r=7, θ=180º or π, so answer is $7e^{i\pi}$ (or any other angle coterminal with π)

   b) r=6, θ=90º, so answer is $6e^{i\frac{\pi}{2}}$ (or any other coterminal angle)

   c) r=5, θ = –37º (or –0.64 rads), so answer is $5e^{-0.64i}$

   d) r=2, θ=45º or π/4, so answer is $2e^{i\frac{\pi}{4}}$

   e) r=12, θ=$\frac{3\pi}{2}$, so answer is $12e^{i\frac{3\pi}{2}}$

2. a) $5e^{i\pi}=5\cos\pi+5i\sin\pi=5(-1)+5i(0)=-5$

   b) $6-6i\sqrt{3}$

   c) $2i$

3. a) $\left(5e^{i\pi}\right)\left(3e^{i\pi}\right)=15e^{i2\pi}=15$

   b) $\left(12e^{-i\frac{\pi}{3}}\right)\left(4e^{i\frac{\pi}{2}}\right)=48e^{i\left(-\frac{\pi}{3}+\frac{\pi}{2}\right)}=48e^{i\frac{\pi}{6}}$

   c) $\left(\sqrt{2}+i\sqrt{2}\right)\left(1-i\sqrt{3}\right)=\left(2e^{i\pi/4}\right)\left(2e^{-i\pi/3}\right)=4e^{i\left(\frac{\pi}{4}-\frac{\pi}{3}\right)}=4e^{-i\left(\frac{\pi}{12}\right)}$

   d) $\left(12e^{-i\frac{\pi}{3}}\right)\div\left(4e^{i\frac{\pi}{2}}\right)=3e^{i\left(-\frac{\pi}{3}-\frac{\pi}{2}\right)}=3e^{-i\frac{5\pi}{6}}$

   e) $\dfrac{\left(\sqrt{2}+i\sqrt{2}\right)}{\left(1-i\sqrt{3}\right)}=\dfrac{\left(2e^{i\pi/4}\right)}{\left(2e^{-i\pi/3}\right)}=1e^{i\left(\frac{\pi}{4}+\frac{\pi}{3}\right)}=1e^{i\left(\frac{7\pi}{12}\right)}$

4. a) $\left(5e^{i\pi}\right)^4 = 625e^{i4\pi} = 625$

b) $\left(12e^{-i\frac{\pi}{3}}\right)^9 = 12^9 e^{-i3\pi} = -12^9$

c) $\left(2e^{i\frac{\pi}{2}}\right)^7 = 128e^{i\frac{7\pi}{2}} = -128i$

d) $\left(\sqrt{2}+i\sqrt{2}\right)^8 = \left(2e^{i\frac{\pi}{4}}\right)^8 = 256e^{i2\pi} = 256$

5. a) $i^{1/4} = \left(1e^{i\frac{\pi}{2}}\right)^{1/4} = 1e^{i\left(\frac{\pi}{8}+\frac{2k\pi}{4}\right)}(k=0,1,2,3)$

$$= 1e^{i\frac{\pi}{8}}(k=0)$$
$$= 1e^{i\left(\frac{\pi}{8}+\frac{2\pi}{4}\right)} = 1e^{i\left(\frac{5\pi}{8}\right)}(k=1)$$
$$= 1e^{i\left(\frac{\pi}{8}+\frac{4\pi}{4}\right)} = 1e^{i\left(\frac{9\pi}{8}\right)}(k=2)$$
$$= 1e^{i\left(\frac{\pi}{8}+\frac{6\pi}{4}\right)} = 1e^{i\left(\frac{13\pi}{8}\right)}(k=3)$$
$$= 1e^{i\frac{\pi}{8}}, 1e^{i\left(\frac{5\pi}{8}\right)}, 1e^{i\left(\frac{9\pi}{8}\right)}, 1e^{i\left(\frac{13\pi}{8}\right)}$$

b) $\left(\sqrt{2}+i\sqrt{2}\right)^{1/4} = \left(2e^{i\frac{\pi}{4}}\right)^{1/4} = 2^{1/4}e^{i\left(\frac{\pi}{16}+\frac{2k\pi}{4}\right)}(k=0,1,2,3)$

$$= 2^{1/4}e^{i\frac{\pi}{16}}(k=0)$$
$$= 2^{1/4}e^{i\left(\frac{\pi}{16}+\frac{2\pi}{4}\right)} = 2^{1/4}e^{i\left(\frac{9\pi}{16}\right)}(k=1)$$
$$= 2^{1/4}e^{i\left(\frac{\pi}{16}+\frac{4\pi}{4}\right)} = 2^{1/4}e^{i\left(\frac{17\pi}{16}\right)}(k=2)$$
$$= 2^{1/4}e^{i\left(\frac{\pi}{16}+\frac{6\pi}{4}\right)} = 2^{1/4}e^{i\left(\frac{25\pi}{16}\right)}(k=3)$$
$$= 2^{1/4}e^{i\frac{\pi}{16}}, 2^{1/4}e^{i\left(\frac{9\pi}{16}\right)}, 2^{1/4}e^{i\left(\frac{17\pi}{16}\right)}, 2^{1/4}e^{i\left(\frac{25\pi}{16}\right)}$$

c) $(-16)^{1/4} = \left(2^4 e^{i\pi}\right)^{1/4} = 2e^{i\left(\frac{\pi}{4}+\frac{2k\pi}{4}\right)} (k = 0,1,2,3)$

$= 2e^{i\frac{\pi}{4}} (k = 0)$

$= 2e^{i\left(\frac{\pi}{4}+\frac{2\pi}{4}\right)} = 2e^{i\left(\frac{3\pi}{4}\right)} (k = 1)$

$= 2e^{i\left(\frac{\pi}{4}+\frac{4\pi}{4}\right)} = 2e^{i\left(\frac{5\pi}{4}\right)} (k = 2)$

$= 2e^{i\left(\frac{\pi}{4}+\frac{6\pi}{4}\right)} = 2e^{i\left(\frac{7\pi}{4}\right)} (k = 3)$

$= 2e^{i\frac{\pi}{4}}, 2e^{i\left(\frac{3\pi}{4}\right)}, 2e^{i\left(\frac{5\pi}{4}\right)}, 2e^{i\left(\frac{7\pi}{4}\right)}$

d) $\left(1-i\sqrt{3}\right)^{1/4} = \left(2e^{-i\frac{\pi}{3}}\right)^{1/4} = 2^{1/4} e^{i\left(-\frac{\pi}{12}+\frac{2k\pi}{4}\right)} (k = 0,1,2,3)$

$= 2^{1/4} e^{-i\frac{\pi}{12}} (k = 0)$

$= 2^{1/4} e^{i\left(-\frac{\pi}{12}+\frac{2\pi}{4}\right)} = 2^{1/4} e^{i\left(\frac{5\pi}{12}\right)} (k = 1)$

$= 2^{1/4} e^{i\left(-\frac{\pi}{12}+\frac{4\pi}{4}\right)} = 2^{1/4} e^{i\left(\frac{11\pi}{12}\right)} (k = 2)$

$= 2^{1/4} e^{i\left(-\frac{\pi}{12}+\frac{6\pi}{4}\right)} = 2^{1/4} e^{i\left(\frac{17\pi}{12}\right)} (k = 3)$

$= 2^{1/4} e^{-i\frac{\pi}{12}}, 2^{1/4} e^{i\left(\frac{5\pi}{12}\right)}, 2^{1/4} e^{i\left(\frac{11\pi}{12}\right)}, 2^{1/4} e^{i\left(\frac{17\pi}{12}\right)}$