

Math 251: Properties of Vectors

Algebraic Properties of Vectors in \mathbb{R}^n :

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors in \mathbb{R}^n and let c and d be scalars. Then

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(d\mathbf{u}) = (cd)\mathbf{u}$
8. $1\mathbf{u} = \mathbf{u}$

Linear Combinations and Coordinates:

Definition

A vector \mathbf{v} is a **linear combination** of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ if there are scalars c_1, c_2, \dots, c_k such that $\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_k\mathbf{v}_k$. The scalars are called the **coefficients** of the linear combination.

In other words, a vector that is a sum of scalar multiples of other vectors is said to be a **linear combination** of those vectors.