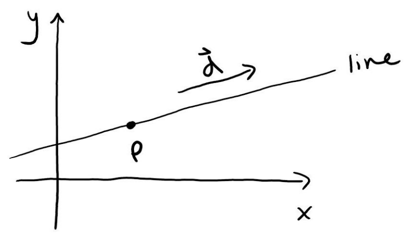
Math 251: Lines and Planes

Lines in $\mathbb{R}^2$:

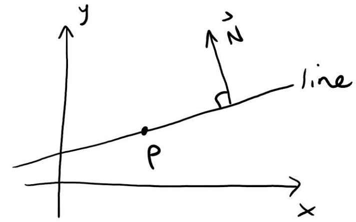


Given: point P and direction vector $\mathbf{d}$

Let: X be some point on the line

Vector Form: $\mathbf{PX} = t\mathbf{d}$

Parametric Form: $\begin{cases} x = p_1 + td_1 \\ y = p_2 + td_2 \end{cases}$



Given: point P and normal vector $\mathbf{N}$

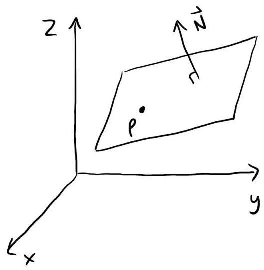
Let: X be some point on the line

Normal Form: $\mathbf{PX} \cdot \mathbf{N} = 0$

General Form: $ax + by = c$

Last modified on September 18, 2018 by Patricia Wrean.

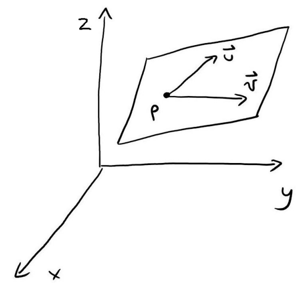
Planes in $\mathbb{R}^3$



Given: point P and normal vector $\mathbf{N}$

Normal Form: $\mathbf{N} \cdot \mathbf{PX} = 0$

General Form: $ax + by + cz = d$

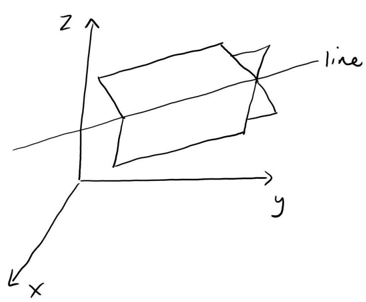


Given: point P and vectors $\mathbf{u}$ and $\mathbf{v}$ in the plane

Vector Form: $\mathbf{PX} = s\mathbf{u} + t\mathbf{v}$

Parametric Form: $$\begin{cases} x = p_1 + su_1 + tv_1 \\ y = p_2 + su_2 + tv_2 \\ z = p_3 + su_3 + tv_3 \end{cases}$$

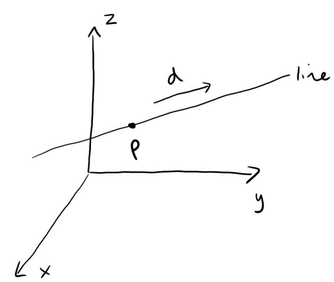
Lines in $\mathbb{R}^3$



Given: intersection of two planes

Normal Form: $$\begin{cases} \mathbf{N_1} \cdot \mathbf{P_1X} = 0 \\ \mathbf{N_2} \cdot \mathbf{P_1X} = 0 \end{cases}$$

General Form: $$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \end{cases}$$



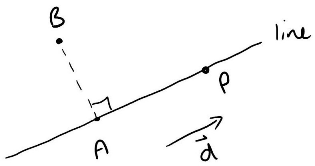
Given: point P and direction vector $\mathbf{d}$

Vector Form: $\mathbf{PX} = t\mathbf{d}$

Parametric Form: $$\begin{cases} x = p_1 + td_1 \\ y = p_2 + td_2 \\ z = p_3 + td_3 \end{cases}$$

Projections

Distance from a point to a line



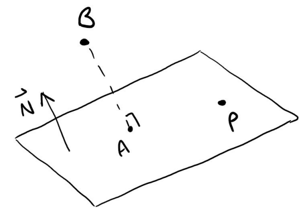
Given points B, P, and vector $\mathbf{d}$

$$\mathbf{PA} = \text{proj}_{\mathbf{d}}(\mathbf{PB})$$

$$\mathbf{AB} = \mathbf{PB} - \mathbf{PA}$$

then distance $= ||\mathbf{AB}||$

Distance from a point to a plane



Given points B, P, and vector $\mathbf{N}$

$$\mathbf{AB} = \text{proj}_{\mathbf{N}}(\mathbf{PB})$$

then distance $= ||\mathbf{AB}||$