Math 251: Homogeneous Systems

Definition: A system is homogeneous if the constant term in each equation is zero.

$$\text{example: } \begin{cases} 3x + y = 0 \\ 2x - 5y = 0 \end{cases}$$

A homogeneous system always has the trivial solution: all variables are equal to zero. As a result, homogeneous systems are always consistent (have at least one solution).

Theorem: For a homogeneous system with M equations and N variables, if $M < N$, then the system has infinitely many solutions.

$$\text{example: } \begin{cases} 2x + 3y + 5z = 0 \\ 4x - y + 2x = 0 \end{cases}$$

This is a homogeneous system with 2 equations in 3 variables $\rightarrow$ infinitely many solutions.