Math 251: Applications of Linear Systems

Curve-fitting

Given 3 points that are not in a straight line, we can find a unique parabola

$$y = ax^2 + bx + c$$

that contains all 3. By plugging the coordinates into x and y, we get three equations in the three variables a, b, and c. (In general: given N points, we can find an equation of a degree N-1 polynomial.)

Allocation of Resources

When you are allocating limited resources, often you have a set of constraints that you need to satisfy.

Balancing Chemical Equations:

The chemical equation

$$2H_2 + O_2 \to 2H_2O$$

is an example of a balanced equation because both sides have 4 H atoms and 2 O atoms. This equation is not unique. We could also have

$$6H_2 + 3O_2 \to 6H_3O$$

But when we look to balance an equation, we want the smallest positive integers that work.

Network Analysis

A network is essentially a map in which there are a finite number of nodes (or junctions or vertices) which are connected by branches. Think of a street map with traffic flow, or an electrical circuit with current flowing through it. At each node, the flow coming in equals the flow going out.

Exercises

1. Given the points $(1, -2)$, $(-1, 8)$, and $(2, -1)$, find the equation $y = ax^2 + bx + c$ that contains them all.

2. NASA is planning a mission to Mars, and their dieticians must design a food supplement that provides 120 mg of magnesium, 220 mg of Vitamin C, and 620 mg of calcium. The three possible ingredients have:

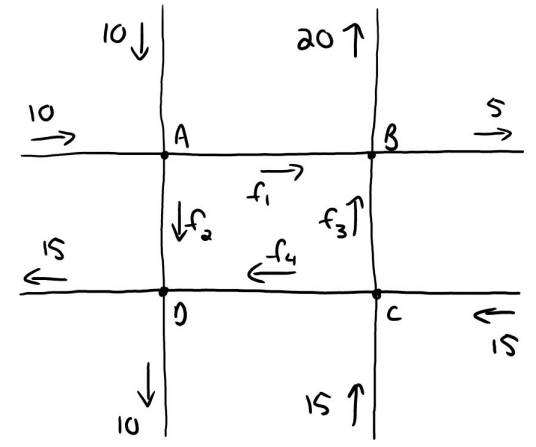
	ingredient 1	ingredient 2	ingredient 3
magnesium	10	30	20
Vitamin C	20	50	30
calcium	60	130	70

   where these numbers are milligrams per unit of food.

   Is it possible to design such a supplement from these ingredients? If so, how much of each ingredient must be used?

3. Balance $NH_3 + O_2 \to N_2 + H_2O$.

4. The downtown core of Gotham City consists of one-way streets, and the traffic flow has been measured at each intersection. For the city block show in the figure below, the numbers represent the average numbers of vehicles per minute entering and leaving intersections A, B, C, and D during business hours.



   (a) Set up and solve a system of linear equations to find the possible flows f_1 to f_4.

   (b) If traffic is regulated on CD such that $f_4 = 10$ vehicles per minute, what will the average flows on the other streets be?

   (c) What are the minimum and maximum possible flows on each street?

5. Find all possible combinations of 20 coins (nickels, dimes, and quarters) that will make exactly \$3.00.