Math 251: Properties of Matrices

Algebraic Properties of Matrices, Addition and Scalar Multiplication:

Let A, B, and C be matrices of the same size and let c and d be scalars. Then

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + 0 = A$
4. $A + (-A) = 0$
5. $c(A + B) = cA + cB$
6. $(c + d)A = cA + dA$
7. $c(dA) = (cd)A$
8. $1A = A$

Properties of Matrix Multiplication:

Let A, B, and C be matrices for which the following products exist, and k be a scalar. Then

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AC + BC$
4. $k(AB) = (kA)B = A(kB)$
5. $I_m A = A = AI_n$ if A is $m \times n$

Properties of the Transpose:

Let A and B be matrices for which the following sums and products exist, k be a scalar, and r be a nonnegative integer. Then

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(kA)^T = k(A^T)$
4. $(AB)^T = B^T A^T$
5. $(A^r)^T = (A^T)^r$

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Properties of the Inverse of a Matrix:

Let A and B be invertible matrices of the same size, c be a non-zero scalar, and n be a nonnegative integer.

Then A^{-1}, cA, AB, A^T, and A^n are also invertible and

1. $(A^{-1})^{-1} = A$
2. $(cA)^{-1} = \frac{1}{c}A^{-1}$
3. $(AB)^{-1} = B^{-1}A^{-1}$
4. $(A^T)^{-1} = (A^{-1})^T$
5. $(A^n)^{-1} = (A^{-1})^n$

The Fundamental Theorem of Invertible Matrices (starter pack):

Let A be an $n \times n$ matrix. The following statements are equivalent:

1. A is invertible.
2. $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ in $\mathbb{R}^n$.
3. $A\mathbf{x} = 0$ has only the trivial solution.
4. The reduced row echelon form of A is I_n.
5. A is a product of elementary matrices.