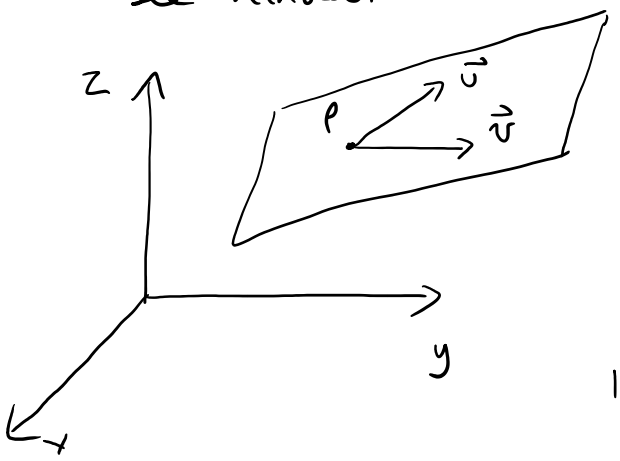
Section 1.3 : cont'd

Tuesday, September 18, 2018 4:21 PM

parametric equations for a plane

see handout



$\vec{u}$ and $\vec{v}$ are not parallel to each other

P is a point on the plane

let $X = (x, y, z)$ be an arbitrary point in the plane

then $\vec{PX} = s\vec{u} + t\vec{v}$

for some scalars/constants s and t

and if you like,

$$\vec{X} - \vec{P} = s\vec{u} + t\vec{v}$$

$$\boxed{\vec{X} = \vec{P} + s\vec{u} + t\vec{v}}$$

example: Find the vector eqn and the parametric equations for the plane containing

$P = (2, 1, 3)$

$Q = (1, 0, 4)$

and $R = (3, 1, -6)$

answer: we want two vectors and a point in the plane

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$\vec{u} = \vec{PR} = \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix}$ and $\vec{v} = \vec{PQ} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$P = (2, 1, 3)$

now $\vec{PX} = s\vec{u} + t\vec{v}$

$$\begin{bmatrix} x-2 \\ y-1 \\ z-3 \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

vector form

note: these equations are not unique - any point and two non-parallel vectors in the plane will do

$$\begin{cases} x = 2 + s - t \\ y = 1 - t \\ z = 3 - 9s + t \end{cases}$$

parametric equations

note: earlier, we found the general equation for this plane to be

$$9x - 8y + z = 13$$

how can we tell that these are equivalent

$$9(2 + s - t) - 8(1 - t) + (3 - 9s + t) = 13$$
$$18 + 9s - 9t - 8 + 8t + 3 - 9s + t = 13$$
$$13 = 13 \checkmark$$

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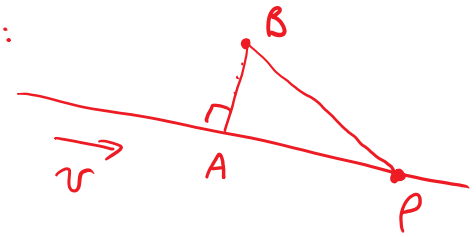
distance from a point to a line in $\mathbb{R}^3$

example: find the distance between the point $B = (1, -1, 2)$ and the line

$$\begin{cases} x = 3 + t \\ y = -2 - 2t \\ z = 4 + 2t \end{cases}$$

answer:



find a point P on the line

$P = (3, -2, 4)$

the direction vector $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$