Section 2.2: cont'd

Tuesday, September 25, 2018 4:28 PM

consider the system

$$\begin{cases} x + 2y = k-1 \\ 2x + (k^2-5)y = 4 \end{cases}$$

where k = constant

find the values of k for which the system has

a) one unique solution
b) infinitely many solutions
c) no solutions

$$\left[\begin{array}{cc|c} 1 & 2 & k-1 \\ 2 & k^2-5 & 4 \end{array}\right] \xrightarrow[R_2 - 2R_1]{} \left[\begin{array}{cc|c} 1 & 2 & k-1 \\ 0 & k^2-9 & 6-2k \end{array}\right]$$

Case 1: what if $k^2 - 9 = 0$?

if $k = -3$:

$$\left[\begin{array}{cc|c} 1 & 2 & -4 \\ 0 & 0 & 12 \end{array}\right]$$

no solutions

if $k = 3$:

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

infinitely many solutions

Case 2: $k^2 - 9 \neq 0$

$k \neq \pm 3$

$$\left[\begin{array}{cc|c} 1 & 2 & k-1 \\ 0 & k^2-9 & 6-2k \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & k-1 \\ 0 & 1 & \frac{6-2k}{k^2-9} \end{array}\right]$$

but $\dfrac{6-2k}{k^2-9} = \dfrac{2(\cancel{3-k})}{(\cancel{k-3})(k+3)} = \dfrac{-2}{k+3}$

$$\left[\begin{array}{cc|c} 1 & 2 & k-1 \\ 0 & 1 & -2/k+3 \end{array}\right]$$

so one solution

to answer the original question:

a) $k \neq \pm 3$ $\quad (k \neq -3$ and $k \neq 3)$

b) $k = 3$

c) $k = -3$

---

definition: two matrices A and B are row-equivalent if there is a sequence of elementary row operations that converts A to B

example #17 from textbook:

show that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$

are row-equivalent

answer: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow[R_2 - 2R_1]{} \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$\downarrow$

$$-\tfrac{1}{2} + \underline{x}\,(1) = 3$$

$$-\tfrac{1}{2}R_1 \begin{bmatrix} -\tfrac{1}{2} & -1 \\ 1 & 0 \end{bmatrix}$$

$$\downarrow$$

$$R_1 + \tfrac{7}{2}R_2 \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix} = 0 \checkmark$$

handout - homogeneous systems

Test 1 is up to here