Section 3.1: cont'd

Wednesday, October 3, 2018   11:19 AM

examples: let $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ 4 & 0 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

a) find $A^T C$

$$A^T C = \underset{3\times 2}{\begin{bmatrix} 2 & 4 \\ 3 & 0 \\ 1 & 5 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}} = \underset{3\times 2}{\begin{bmatrix} 4 & 14 \\ 6 & 3 \\ 2 & 16 \end{bmatrix}}$$

b) $AB + C^2$

$$AB + C^2 = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 4 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 9 \\ 19 & 21 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 14 \\ 19 & 30 \end{bmatrix}$$

c) CB

$$CB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 4 & 0 \\ 3 & 1 \end{bmatrix} = \text{undefined}$$

example: let $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$

Find a) AB

b) BA

a) $AB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 23 \end{bmatrix}$

b) $BA = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 1 & 2 & 3 \\ 6 & 12 & 18 \end{bmatrix}$

note: there is another way to multiply a matrix by a column vector:

example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 17 \\ 41 \end{bmatrix}$

block multiplication:

example: calculate AB if

$$A = \left[\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ \hline 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & 5 \end{array}\right] \quad \text{and} \quad B = \left[\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 2 & 1 & 1 & 2 \\ 4 & 7 & 3 & 4 \end{array}\right]$$

$$A = \left[\begin{array}{c|c} A_1 & A_2 = 0 \\ \hline A_3 = I_2 & A_4 \end{array}\right] \qquad B = \left[\begin{array}{c|c} B_1 = 0 & B_2 = I_2 \\ \hline B_3 & B_4 \end{array}\right]$$

then $AB = \left[\begin{array}{c|c} 0 & \overset{= A_1}{A_1 I_2 + 0} \\ \hline A_4 B_3 & I_2 + A_4 B_4 \end{array}\right] = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 14 & 11 & 11 & 14 \\ 18 & 34 & 14 & 19 \end{bmatrix}$