Section 3.2: Matrix Algebra

Wednesday, October 3, 2018 11:55 AM

handout: look at first group: Algebraic pptires: addition and scalar mult

definition: $c_1 A_1 + c_2 A_2 + \dots c_n A_n$

is a linear combination of the matrices $A_1, A_2, A_3, \dots A_n$

example: Is $B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ a LC of

$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?

answer: $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{aligned} 2 &= c_2 + c_3 \\ 1 &= c_1 + c_3 \\ -3 &= -c_1 + c_3 \\ 2 &= c_2 + c_3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & -3 \\ 0 & 1 & 1 & 2 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$c_1 = 2$

$c_2 = 3$

$c_3 = -1$

so $B = 2A_1 + 3A_2 - A_3$, and $\boxed{\text{yes}}$

definition: the <u>span</u> of a set of $m \times N$ matrices is the set of all linear combinations of the matrices

so, from our previous example,

B is in span(A_1, A_2, A_3)

definition: matrices $A_1, A_2, A_3, \ldots A_n$ are linearly independent (LI) if

$$c_1 A_1 + c_2 A_2 + \ldots c_n A_n = 0$$

only has the trivial solution

$$c_1 = c_2 = c_3 = \ldots c_n = 0$$

otherwise they are linearly dependent (LD).