Review for Test 1:

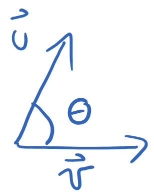
Thursday, October 4, 2018 12:52 PM

A few ideas:

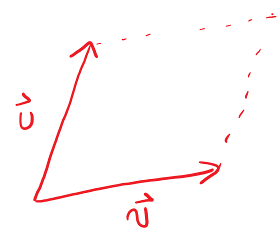
① if $\vec{u} \perp \vec{v}$, then $\vec{u} \cdot \vec{v} = 0$

② if $\vec{u} \parallel \vec{v}$, then $\vec{u} = c\,\vec{v}$
where c is a scalar / constant

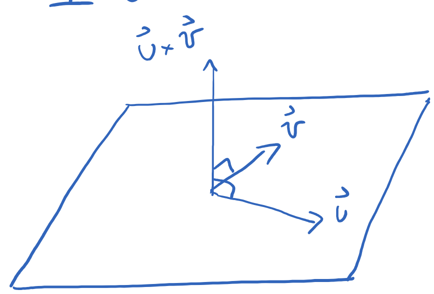
③ $\vec{u} \cdot \vec{v} = \|\vec{u}\|\,\|\vec{v}\| \cos\theta$ where $0 \le \theta \le 180°$



④ $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\,\|\vec{v}\| \sin\theta$
= area of parallelogram



⑤ $(\vec{u} \times \vec{v}) \perp \vec{u}$ and $(\vec{u} \times \vec{v}) \perp \vec{v}$



note: $\vec{u}$ and $\vec{v}$ do not have to be perpendicular

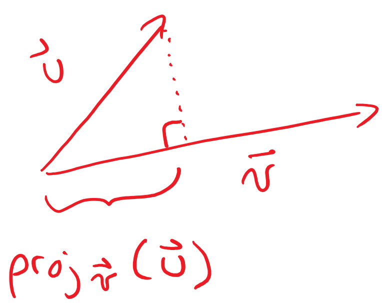
6. unit vector – has length / magnitude / norm of one

how do you get one? take the vector and divide by the magnitude

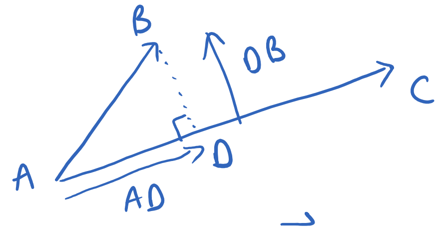
7. in $\mathbb{R}^3$ $\quad \|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$ for $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

note also: $\|\vec{u}\|^2 = \vec{u} \cdot \vec{u}$

8. projections



find perpendicular component by subtraction

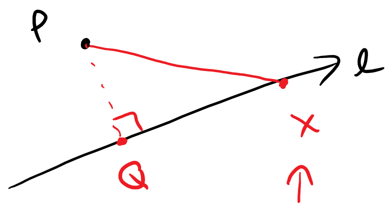


$\vec{AD} = \text{proj}_{\vec{AC}}(\vec{AB})$

$\vec{AD} + \vec{DB} = \vec{AB}$

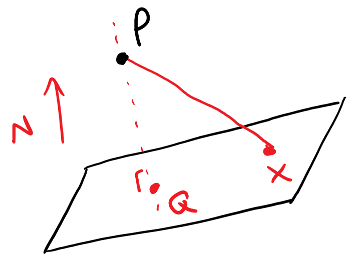
$\vec{DB} = \vec{AB} - \vec{AD}$

9. distance from point to line / plane



find some point on the line

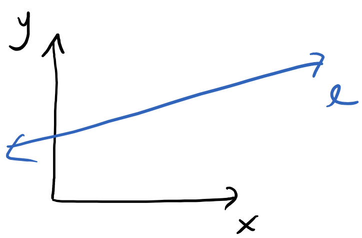
proj will be $\vec{XQ}$
need subtraction to get $\vec{PQ}$



proj will be $\vec{PQ}$

⑩ lines and planes

lines in $\mathbb{R}^2$



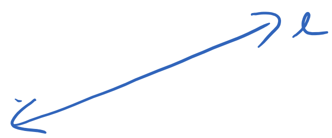
need:

ⓐ two points

ⓑ one point and a direction vector

ⓒ one point and a normal vector

lines in $\mathbb{R}^3$



need:

ⓐ two points

ⓑ one point and a direction vector

ⓒ intersection of two planes

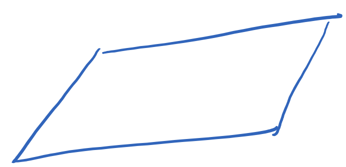
general form

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \end{cases}$$

parametric form

$x = p + td$

planes in $\mathbb{R}^3$



need:

ⓐ three points

ⓑ one point and two vectors in the plane

ⓒ one point and a normal

general form

$$ax + by + cz = d$$

parametric form

$x = p + su + tv$

parametric form

$$\begin{cases} x = p_1 + td_1 \\ y = p_2 + td_2 \\ z = p_3 + td_3 \end{cases}$$

parametric form

$$\begin{cases} x = p_1 + su_1 + tv_1 \\ y = p_2 + su_2 + tv_2 \\ z = p_3 + su_3 + tv_3 \end{cases}$$

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11. systems of equations

RREF - see notes for rules

free variable - any variable that does not have a leading one

$\Rightarrow$ infinitely many solutions

$$\begin{array}{c} \begin{matrix} x & y & z & & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

z is free variable

let $z = t$

solution : $\begin{cases} x = 3 - 2t \\ y = 1 - t \\ z = t \end{cases}$