Section 3.2: Cont'd:

Tuesday, October 9, 2018 4:29 PM

example: Are $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

linearly independent?

answer: $c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

where

$$\begin{aligned} c_2 + c_3 &= 0 && \text{top left coord} \\ c_1 + c_3 &= 0 && \text{top right} \\ -c_1 + c_3 &= 0 && \text{bottom left} \\ c_2 + c_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

which has solution $c_1 = 0$, $c_2 = 0$, $c_3 = 0$

is the unique solution

so A_1, A_2, and A_3 are LI

we looked at properties of matrix multiplication on handout

WARNING:

$$(A+B)^2 = (A+B)(A+B)$$

$$= A^2 + \underbrace{AB + BA}_{} + B^2$$ ok so far

but cannot assume that AB is equal to BA

$$AB \neq BA$$ in general for matrices

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back to transpose:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

and we looked at the properties of the transpose on the handout, but noted that

$$(AB)^T = B^T A^T$$