Section 3.3: cont'd

Thursday, October 11, 2018 12:56 PM

Test #2 on: Friday, Nov 2

covers sections 2.3 to 3.6, inclusive

formula sheet: if you need the projection operator, I will provide it

if you need the rotation matrix, then I will provide it <u>unless</u> I ask you

to derive it from scratch

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look at matrix properties handout: page 2

Properties of the Inverse

why is #3 true? $(AB)^{-1} = B^{-1}A^{-1}$

take
$$(AB)(B^{-1}A^{-1}) = ABB^{-1}A^{-1}$$
$$= A(BB^{-1})A^{-1}$$
$$= A\,I_n\,A^{-1}$$
$$= AA^{-1}$$
$$= I$$

$$\begin{aligned}(B^{-1}A^{-1})(AB) &= B^{-1}A^{-1}AB \\ &= B^{-1}\,I\,B \\ &= B^{-1}B \\ &= I\end{aligned}$$

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so $(AB)^{-1} = B^{-1}A^{-1}$

For matrices larger than 2×2, we use Gauss-Jordan method to find A^{-1}.

$$[A \mid I] \xrightarrow{RREF} [I \mid A^{-1}]$$

example: let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -5 \\ -1 & -2 & 0 \end{bmatrix}$. Find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -5 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+R_1]{R_2-3R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -3 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\downarrow R_1-2R_2,\ -R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -2 & 3 \\ 0 & 1 & 0 & -5 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right] \xleftarrow[R_2+2R_3]{R_1-3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 7 & -2 & 0 \\ 0 & 1 & -2 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 10 & -2 & 3 \\ -5 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

check: $AA^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -5 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 10 & -2 & 3 \\ -5 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

now use A^{-1} to solve:
$$\begin{cases} x + 2y - z = 2 \\ 3x + 7y - 5z = 5 \\ -x - 2y = 1 \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -5 \\ -1 & -2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}}_{\vec{b}}$$

$$A\vec{x} = \vec{b}$$
$$A^{-1}A\vec{x} = A^{-1}\vec{b}$$
$$\vec{x} = A^{-1}\vec{b}$$

$$= \begin{bmatrix} 10 & -2 & 3 \\ -5 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ -7 \\ -3 \end{bmatrix} \leftarrow 10(2) + (-2)(5) + (3)(1)$$

solution to the system is $x = 13$, $y = -7$, $z = -3$

definition: An elementary matrix is a matrix that can be obtained by performing one elementary row operation on an identity matrix.

If E is an elementary matrix corresponding to a row operation:

$$I \xrightarrow{\text{row}} E \qquad \text{then } B = EA$$

$$\left.\begin{array}{l} I \xrightarrow[\text{operation}]{\text{row}} E \\ A \xrightarrow[\text{row op}]{\text{same}} B \end{array}\right\} \text{ then } B = EA$$

examples:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2 \Leftrightarrow R_3]{} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = E$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 6 & 8 \end{bmatrix} \xrightarrow[R_2 \Leftrightarrow R_3]{} \begin{bmatrix} 2 & 3 \\ 6 & 8 \\ 4 & 1 \end{bmatrix} = B$$

then $$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 6 & 8 \\ 4 & 1 \end{bmatrix}$$