Review for Test #3:

Tuesday, November 27, 2018 4:42 PM

Complex numbers:

$z = a + bi$ (rectangular)

$$\left.\begin{aligned} z &= r(\cos\theta + i\sin\theta) && \text{(trig)} \\ &= re^{i\theta} && \text{(Euler/exponential)} \\ &= r\angle\theta && \text{(phasor)} \end{aligned}\right\} \text{polar}$$

- addition/subtraction $(3+5i) + (2-7i)$
- multiplication/division

$$(3+5i)(2-7i)$$

$$\frac{3+5i}{2-7i}\left(\frac{2+7i}{2+7i}\right)$$

$$\frac{2e^{i\pi}}{4e^{-5\pi/6\, i}}$$

- to find complex roots:

if $z^n = r\angle\theta$

then $z = r^{1/n} \underline{\angle \frac{\theta}{n} + \frac{k360^\circ}{n}}$ for $k = \underbrace{0, 1, 2, \ldots n-1}_{n \text{ of them}}$

note: eigenvalues/eigenvectors can be complex

Chapter 4:

Determinants of square matrices

3 special cases:

2×2 $\quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

3×3 $\quad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ cross product method

triangular matrix of any size
- product of main diagonal

in general:

method of minors / cofactor expansion

expand along top row $\rightarrow$ $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = +a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix}$ etc

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Cofactor definition of A^{-1}

- I did not give you homework on this so I will not test it

Cramer's Rule: if $A\vec{x} = \vec{b}$ where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$

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then $$x_i = \frac{\det(A_i(\vec{b}))}{\det(A)}$$

the matrix obtained by replacing column i of A by $\vec{b}$

Eigenvalues: def: if $A\vec{v} = \lambda\vec{v}$ for $v \neq \vec{0}$
then λ is an eigenvalue and
$\vec{v}$ is its eigenvector

to find λ: solve $\det(A - \lambda I) = 0$

- you will get a polynomial in λ
  - factor it

- the power on each factor gives the algebraic multiplicity

for each λ, to find the eigenvector,

solve $(A - \lambda I)\vec{x} = \vec{0}$

RREF

- the dimension of the eigenspace
  = number of free variables in RREF
  = number of eigenvectors for that λ
  = geometric multiplicity

note: eigenvalues and eigenvectors may be complex

Diagonalization: $A = PDP^{-1}$

where $D = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$ λ_n has eigenvector $\vec{v}_n$

and $P = \left[\vec{v}_1 \mid \vec{v}_2 \mid \cdots \mid \vec{v}_n \right]$

A is diagonalizable if and only if alg mult = geom mult for every λ

convenient application of $A = PDP^{-1}$

$$A^m = PD^mP^{-1} = P \begin{bmatrix} \lambda_1^m & & & 0 \\ & \lambda_2^m & & \\ & & \ddots & \\ 0 & & & \lambda^m \end{bmatrix} P^{-1}$$