Review for Test 3:

Thursday, November 29, 2018 12:59 PM

Chapter 5:

- def: $\vec{u}$ and $\vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$
- every orthogonal set of vectors is LI
- if $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \dots \vec{v}_k\}$ is an orthogonal basis of W

and $\vec{v}$ is in W, then

$$[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} \frac{\vec{v} \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \\ \frac{\vec{v} \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2} \\ \text{etc} \end{bmatrix}$$

recall: if $\mathcal{B}$ is not orthogonal, find $[\vec{v}]_{\mathcal{B}}$ by solving

$$\left[\vec{v}_1 \mid \vec{v}_2 \mid v_3 \mid \dots \; \vec{v} \right]$$ and RREF

def: $\vec{u}$ and $\vec{v}$ are orthonormal if they are orthogonal unit vectors

for $\vec{u}$, $\frac{\vec{u}}{\|\vec{u}\|}$ is its unit vector

def: an orthogonal matrix has orthonormal columns

properties: if Q is orthogonal

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1) $Q^{-1} = Q^T$

2) all eigenvalues satisfy $|\lambda| = 1$

3) $\|Q\vec{x}\| = \|\vec{x}\|$ and $Q\vec{x} \cdot Q\vec{y} = \vec{x} \cdot \vec{y}$ for all $\vec{x}, \vec{y}$

- def: the orthogonal complement of a subspace W is

$$W^{\perp} = \{ \vec{v} \mid \vec{v} \cdot \vec{w} = 0 \text{ for all } \vec{w} \text{ in } W \}$$

- for any matrix A:

$$[\text{row}(A)]^{\perp} = \text{null}(A)$$

$$[\text{col}(A)]^{\perp} = \text{null}(A^T)$$

to find null (A), solve $A\vec{x} = \vec{0}$

$[A \mid 0] \xrightarrow{RREF}$

- if $\{\vec{w}_1, \vec{w}_2, \vec{w}_3 \ldots \vec{w}_k\}$ is an orthogonal basis of W, then

$$\text{proj}_W(\vec{v}) = \text{proj}_{\vec{w}_1}(\vec{v}) + \text{proj}_{\vec{w}_2}(\vec{v}) + \ldots \text{proj}_{\vec{w}_k}(\vec{v}) \quad \in \text{in } W$$

$$\text{perp}_W(\vec{v}) = \vec{v} - \text{proj}_W(\vec{v}) \quad \in \text{in } W_{\perp}$$

(I will give you the projection vector)

- Gram Schmidt - turn any basis into orthogonal basis so that projection onto subspace can be defined start with $\{\vec{x}_1, \vec{x}_2, \vec{x}_3 \dots \vec{x}_k\}$

$\vec{v}_1 = \vec{x}_1$

$\vec{v}_2 = \vec{x}_2 - \text{proj}_{\vec{v}_1}(\vec{x}_2)$

$\vdots$

$\vec{v}_k = \vec{x}_k - \text{proj}_{\vec{v}_1}(\vec{x}_k) - \text{proj}_{\vec{v}_2}(\vec{x}_k) \dots - \text{proj}_{v_{k-1}}(\vec{x}_k)$

- $A = QR$ where Q is an orthogonal matrix (orthonormal columns)
  R is an invertible upper triangular matrix

we can get a QR factorization for A if A has LI columns

→ to find Q: apply Gram-Schmidt to the columns of A, then scale to get orthonormal vectors

→ these are the columns of Q

→ $R = Q^T A$