Section 3.2: Matrix Algebra

Wednesday, October 13, 2021 3:30 PM

we looked at Algebraic properties from handout

definition:

$$c_1 A_1 + c_2 A_2 + \dots \quad c_n A_n$$

is a linear combination of matrices $A_1, A_2, A_3 \dots A_n$ provided that all matrices are the same size

example: Is $B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ a LC (linear combo)

of $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?

answer: $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{aligned} 2 &= c_2 + c_3 \\ 1 &= c_1 + c_3 \\ -3 &= -c_1 + c_3 \\ 2 &= c_2 + c_3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & -3 \\ 0 & 1 & 1 & 2 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$c_1 = 2$

$c_2 = 3$

$c_3 = -1$

so $B = 2A_1 + 3A_2 - A_3$

so $\boxed{\text{yes}}$

definition: the span of a set of $M \times N$ matrices is the set of all linear combinations of the matrices

so, from our previous example

B is in $\text{span}(A_1, A_2, A_3)$

definition: matrices $A_1, A_2, A_3, \dots A_n$ are linearly independent (LI) if

$$c_1 A_1 + c_2 A_2 + \dots c_n A_n = 0$$

only has the trivial solution
$$c_1 = c_2 = c_3 = \dots c_n = 0$$

otherwise, they are linearly dependent, LD

example: Are $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ linearly independent?

answer: $c_1 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0$

where
$$\begin{cases} c_2 + c_3 = 0 \\ c_1 + c_3 = 0 \\ -c_1 + c_3 = 0 \end{cases}$$

$$\left\{\begin{array}{rrrl} c_1 & & +c_3 & = 0 \\ -c_1 & & +c_3 & = 0 \\ & c_2 & +c_3 & = 0 \end{array}\right.$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So the solution $c_1 = 0$
$c_2 = 0$
$c_3 = 0$
is the unique solution

So $A_1, A_2,$ and A_3 are LI

we looked at Properties of Matrix Properties on Handout

Warning:

recall $AB \neq BA$ in general

So $(A+B)^2 = (A+B)(A+B)$

$$= A^2 + \underbrace{AB + BA}_{} + B^2 \quad \text{ok so far}$$

but you cannot assume that $AB = BA$ here

we looked at the Properties of the Transpose

- only really noteworthy one:

4 $(AB)^T = B^T A^T$

for example, if A is 3×2
B is 2×4

then AB is 3×4
$3 \times 2 \quad 2 \times 4$

but $A^T B^T$ undefined
$2 \times 3 \quad 4 \times 2$

but also $B^T A^T$ is defined
$4 \times 2 \quad 2 \times 3$