Review of 5.3 to 7.3

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Section 5.3

- Gram Schmidt – turn any basis into orthogonal basis so that projection onto subspace can be defined

start with $\{\vec{x}_1, \vec{x}_2, \vec{x}_3 \dots \vec{x}_k\}$

$\vec{v}_1 = \vec{x}_1$

$\vec{v}_2 = \vec{x}_2 - \text{proj}_{\vec{v}_1}(\vec{x}_2)$

$\vdots$

$\vec{v}_k = \vec{x}_k - \text{proj}_{\vec{v}_1}(\vec{x}_k) - \text{proj}_{\vec{v}_2}(\vec{x}_k) \dots - \text{proj}_{v_{k-1}}(\vec{x}_k)$

$A = QR$ where Q is an orthogonal matrix (orthonormal columns)
R is an invertible upper triangular matrix

we can get a QR factorization for A if A has LI columns

→ to find Q: apply Gram-Schmidt to the columns of A, then scale to get orthonormal vectors

→ these are the columns of Q

→ $R = Q^T A$

section 5.4:

for symmetric matrices $(A^T = A)$, we can orthogonally diagonalize

to get $A = QDQ^T$ where

Q = orthogonal matrix (orthonormal columns)

D = diagonal matrix

$$D = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

where the λ_i are the eigenvalues of A

$$Q = \left[\vec{q}_1 \,\middle|\, \vec{q}_2 \,\middle|\, \cdots \, \vec{q}_n \right]$$

where $\vec{q}_i$ is an eigenvector for λ_i

and $\{\vec{q}_1, \vec{q}_2 \ldots \vec{q}_n\}$ is orthonormal

- eigenvectors for distinct eigenvalues are $\perp$
- for repeated eigenvalues (alg mult > 1)
  - we use Gram Schmidt to orthogonalize the eigenvectors for that eigenvalue
- scale to get unit vectors

Section 7.3:

For an incensistent system $A\vec{x} = \vec{b}$
(no solution)

$A\vec{x}_{LS} = \text{proj}_{Col(A)} \vec{b}$

- $\vec{x}_{LS} = (A^TA)^{-1} A^T \vec{b}$ ← will be given

$\|\vec{b} - A\vec{x}_{LS}\|$ = least squares error

- to find the least-squares linear fit for (x_1, y_1), (x_2, y_2), ... (x_n, y_n)

line: $y = mx + b$

plug in points:

$$\begin{cases} y_1 = mx_1 + b \\ y_2 = mx_2 + b \\ \vdots \\ y_n = mx_n + b \end{cases}$$

system of equations in m and b

this yields a matrix equation

$$\underbrace{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ & 1 \\ x_n & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} m \\ b \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\vec{b}}$$

- solve for $\vec{x}_{LS}$ to read off m and b