

Review for Test 1:

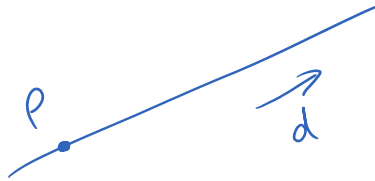
Wednesday, September 28, 2022 12:45 PM

Section 1.3: Lines and Planes

two ways to represent a line in \mathbb{R}^3 :

①

point plus a direction vector



any point on line \vec{x} has

vector form

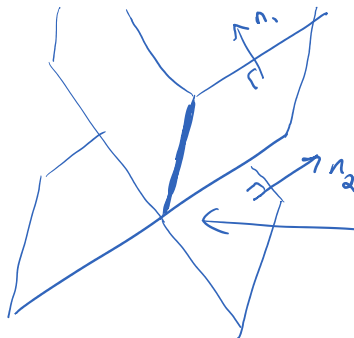
$$\vec{x} = \vec{p} + t\vec{d}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + t \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\begin{cases} x = p_1 + td_1 \\ y = p_2 + td_2 \\ z = p_3 + td_3 \end{cases}$$

parametric form

②



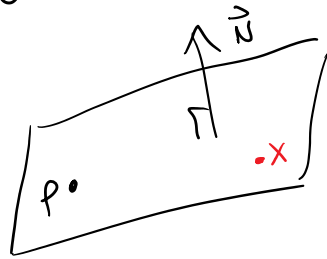
line is intersection of two planes

general form

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \end{cases}$$

two ways to represent a plane in \mathbb{R}^3 :

①



a point and a normal \vec{N}

general form

$$ax + by + cz = d$$

where $\vec{N} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

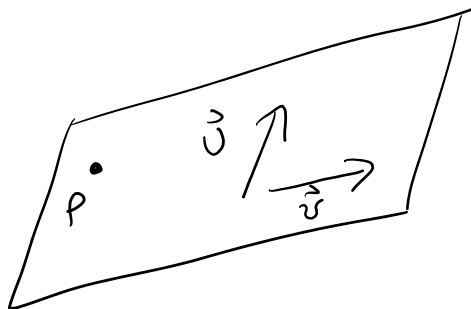
plug in point p to find d

normal form:

$$\vec{px} \cdot \vec{N} = 0$$

$$\begin{bmatrix} x - p_1 \\ y - p_2 \\ z - p_3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

②



a point p and two vectors \vec{u} and \vec{v} (\vec{u} is not \parallel with \vec{v})

$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$$

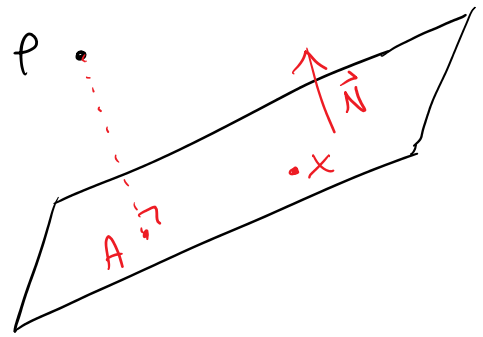
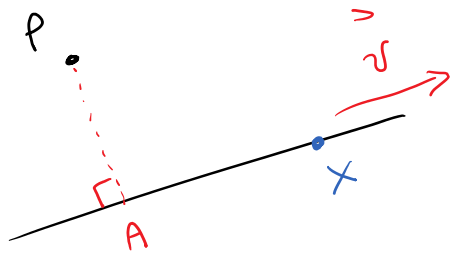
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + s \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

vector form

$$\left\{ \begin{array}{l} x = p_1 + su_1 + tv_1 \\ \text{etc} \end{array} \right.$$

parametric form

distance from point to line / plane



pick some other
point on line
x

$$\vec{PA} = \text{proj}_{\vec{v}} (\vec{PX})$$

know \vec{PX} and \vec{v}

$$\vec{AX} = \text{proj}_{\vec{v}} (\vec{PX})$$

vector subtraction
gives \vec{PA}