Section 1.1: The Geometry and Algebra of Vectors

Tuesday, September 05, 2023 4:04 PM

vectors in the plane:

- a vector is a directed line segment that corresponds to a displacement from one point A to another point B

in general, A and B have coordinates

(x_1, y_1) and (x_2, y_2) for 2D vectors

↑ ↗

points have round brackets

then vector $\overrightarrow{AB}$ can be written as

$$[x_2 - x_1, \; y_2 - y_1]$$

↗

written as a row vector $[3, -2]$

can also write as a column vector

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

note: in WeBWorK, angle brackets $\langle 3, -2 \rangle$ are another common notation

also, $\overrightarrow{AB}$ is <u>from</u> A <u>to</u> B

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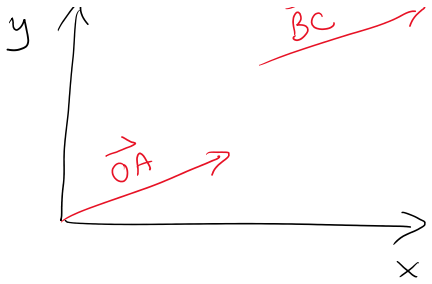
equality of vectors:

these two



these two vectors are equal

vector $\vec{OA}$ is said to be in standard position since its tail is at the origin

one convention: $\vec{OA} = \vec{A}$

O for origin

notation: in 2D:

$\vec{0} = [0, 0]$

the zero vector

this is hard to picture! $\vec{0A}$ is a perfectly good vector!

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the set of all vectors with two components is written

$\mathbb{R}^2$ (pronounced "R two")

three components is $\mathbb{R}^3$

n components is $\mathbb{R}^n$

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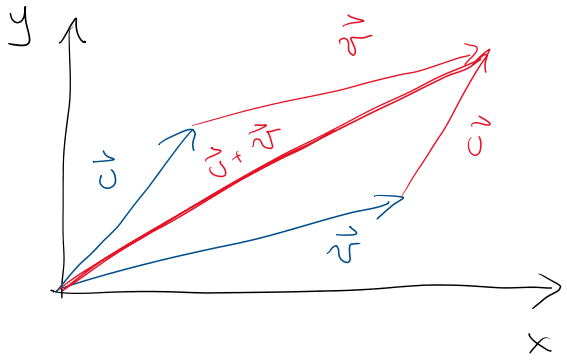
vector addition:



$$\vec{u} + \vec{v} = [u_1 + v_1 \;,\; u_2 + v_2]$$

note: order doesn't matter

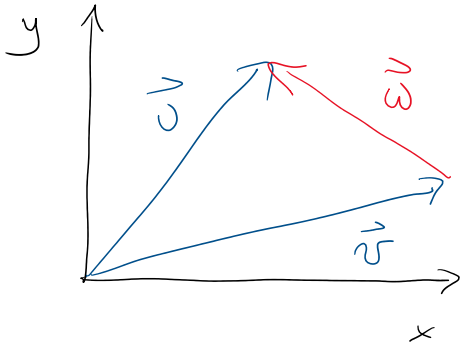
scaling a vector:

$\vec{v}$ is a vector

c is a constant (scalar)

if $\vec{v} = [v_1, v_2]$, then $c\vec{v} = [cv_1, cv_2]$

vector subtraction:



$$\vec{v} + \vec{\omega} = \vec{u}$$

$$\vec{\omega} = \vec{u} - \vec{v}$$

linear combination of vectors: definition is on handout

example: $\vec{w} = -3\begin{bmatrix}1\\1\end{bmatrix} + 2\begin{bmatrix}0\\2\end{bmatrix}$

coefficients

then $\vec{w}$ is a linear combo of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0\\2\end{bmatrix}$

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omit: Binary Vectors and Modular Arithmetic